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A Novel Numerical Experiments and Investigation on Energy Dissipation over Stepped Chute

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Abstract

Stepped spillways are a typical configuration that is used for releasing water and floods from an embankment dam. Moreover, the stepped chute helps to enhance energy dissipation. Stepped spillways have been investigated for more than three decades and in more advanced studies the flow over spillway has been investigated by numerical methods. In this paper, three models of stepped chutes, including flat steps, quarter cut steps, and Half- cut steps were numerically simulated using InterFOAM solver in the OpenFOAM package. The volume of fluid (VOF) method was employed to determine the position of the free surface, and the Realizable k-ε was used as turbulence model. The results indicate that, among all the configurations, the half cut steps were the most effective, such that the energy dissipation was improved up to 8%. Moreover, it was observed that, when discharges increased, the energy dissipation rate decreased as it entered a phase where the impact of the shape of the steps was little.

Keywords: Stepped spillway; InterFOAM; VOF; Realizable k-ε model; Energy dissipation.

الملخص العربي

تعتبر المفائض المدرجة من المنشآت المهمة في السدود حيث يستخدم الدرج لتهدئة وتبديد طاقة الماء الفائض عن المنسوب التصميمي لخزان السد وكذلك لتصغير حجم أحواض التسكين المستخدمة في اسفل السد. ركزت الدراسات السابقة للمفائض المدرجة على التجارب المعملية والتي تعتبر باهظة الثمن وقد تؤدي إلى مشكلات مختلفة تتعلق بأجهزة القياس. مع التطور التكنولوجي اصبحت النمذجة العددية أسرع وأرخص بكثير من التجارب المعملية ، كما أنها تتيح تطبيق التغييرات في تصميم النماذج بسهولة. الهدف من هذا البحث هو دراسة عدة نماذج من المفائض المدرجة بواسطة النمذجة العددية، النموذج الأول يتكون من الدرج العادي، و النموذج الثاني يحتوي على درج ربع مقطوع ، والنموذج الاخير يتكون من درج نصف مقطوع. تم استحدام كود **InterFOAM** الموجود في برنامج **OpenFOAM** في عملية المحاكاة العددية. لتحديد السطح الحر للمياه فوق المفيض المدرج تم استخدام طريقة حجم السائل (**VOF**) ، كما استخدم نموذج **Realizable k-ε** لمحاكاة التدفق المضطرب فوق الدرج. أظهرت النتائج أن الدرج نصف المقطوع هو الأكثر فعالية وأدى الى زيادة في تبديد الطاقة بنسبة تصل إلى 8٪، كما لوحظ أنه عند زيادة تدفق الماء فوق المفيض ينخفض معدل تبديد الطاقة بحيث يصل الى مرحلة يكون تأثير شكل الدرج فيها ضئيل جداً.

1 INTRODUCTION

The spillways structure should have enough discharge and floodwaters capability to prevent the dam from being damaged. At the time of releasing surplus water, the spillway is at risk of scouring and erosion due to high velocity flows emanating from the weirs [1, 2]. To avoid scour and erosion hazards and to protect the structures, hydraulic jump stilling basins are made [3]. The stilling basins are a common type of energy dissipater structures. A good energy dissipater design can reduce water velocity without destroying the hydraulic structures [4]. Flow regime change through a hydraulic jump is the main concept of energy dissipation [5] . The design of the stilling basin can, for instance, vary from the basic to complex structures with rows of chute boxes, baffle piers, and dentate [6]. It is important to keep the size of the stilling basin (at the foot of the spillway) as short as possible [7, 8, 9]. To this end, one solution is a stepped spillway. A stepped spillway is defined as a spillway with steps on the chute and it is utilized to improve energy dissipation [9]. The water flows over the steppes before reaching the end of the chute. In this process, the velocity of the flow reduces. Hence, the downstream energy dissipater basin can be eliminated and thus the construction costs will reduced [4].

Many studies have shown that the energy losses reduce with an increment of discharges as it enters a phase where the impact of the steps is exceptionally little, and a stepped chute seems alike to a smooth one [10, 11, 12]. Chanson [3] and Boes and Hager [13] establish that the energy dissipation does increase with the given height of the dam and its air concentration. Chinnarasri and Wongwises [14] studied the energy dissipation by investigation the impact of changes in the steps geometry, including configuration with horizontal steps, inclined steps, and steps with end sills. They observed that the energy dissipation increases by using steps with end sills for slope 45°. Felder and Chanson [15] conducted experiments on pooled and flat stepped chutes with gradients of 8.9° and 26.6°. The results showed that energy dissipation in the pooled stepped spillway is larger than in the flat stepped spillway with a smaller slope. In the pooled stepped spillway, the mean air concentration is larger than that in the flat stepped spillway. Zhang [16] has provided an investigation on the energy dissipation for stepped spillways with various types of uniform steps. The results showed that the energy dissipation was similar in all except chamfered chute at large discharges.

In addition to the above experimental studies, the flow hydraulics on the stepped spillway has been numerically examined in order to lower cost and save time. In the common numerical studies of stepped spillways, the flow over the stepped chute was studied by the Volume of Fluid (VOF) model with turbulence closures of the k-ε family. For instance, Chen et al. [7] simulated the WES standard stepped spillway profile by utilizing St k-ε model and VOF. They found that the features of the velocities and pressure distributions of the numerical and physical models were comparable. Cheng et al. [17] have used St k-ε model and VOF to analyze air in the water flow over steps of downstream face. They concluded that with the increased discharge, the void fraction is decreased. Qian et al. [18] have simulated water flow over stepped spillway using four turbulence models: SST k-ω model, v2-f model, Realizable k-ε model and LES model. They have validated the numerical models against experimental data in three aspects: mean velocity, the spanwise vortices and boundary layer thickness in streamwise direction. They observed that realizable k-ε was the most effective turbulence model in the simulations. Kositgittiwong et al. [8] have studied flow velocity distribution along a stepped chute using five turbulence models (St k- ε, RNG k- ε, Realizable k- ε, St k-ω and k-ω-SST) with VOF. They found that Realizable k- ε provided slightly better results with the lowest deviation in terms of the root mean square error. Cheng et al. [19] have compared the 2-D simulation outputs with three physical models for characterizing the flow over stepped spillways. The numerical model was based on the VOF model and RNG k-ε model for simulations. The simulations outcomes were in acceptable concurrence with the experimental data for velocity profiles. Shahheydari et al. [9] first validated the numerical model with experimental data. After that, the rates of energy dissipation were studied for several spillway layouts. They found that the discharge coefficient has an inverse relationship with the energy dissipation rate, and also observed that the energy dissipation rate increased when the spillway slope decreased. Tabari and Tavakoli [20] studied the impacts of the stepped spillway geometry with slope 45 on the energy dissipation using a numerical model of Flow-3D and concluded that the energy losses increased with reducing discharges and it reduced with the number of steps increments and their height diminishes. Hekmatzadeh et al [21] simulated flow on a stepped spillway with a mix of plain and a pooled step. The results indicated that if the spillway slope is larger than 14°, the plain stepped chute is more compelling than pooled stepped chute with respect to energy dissipation. The recent studies which focused the non-aerated region over stepped spillway with ogee-crest weir have provided good comparisons of water depths ,and velocity profiles and development of the boundary layer [22, 23, 24]. Wan et al [25] used VOF and realizable k- ε model to define the inception point of air entrainment in different stepped spillways, including plain steps, pooled steps, and round steps. The results indicated that as the step height decreased, the air entrainment decreased, and with respect to aeration, the round stepped spillway was the optimum design.

In this numerical work, an attempt to improve the energy dissipation employing novel step shapes which are quarter- cut steps and half-cut steps. Numerical experiments were performed by the InterFOAM solver in the OpenFOAM software, applying the VOF method and the realizable k-ε turbulent model.

2 MATERIALS AND METHODS

Numerical modeling is a generally used method achieved with the help of a computer to solve the Navier–Stokes equation, which is based upon mass conservation, energy, and momentum. CFD software is used to simulate the stepped chute configurations. Three kinds of stepped spillway configurations with various step dimensions and configurations utilized are listed in Table 1. In order to investigate the rate of energy dissipation performance for all configurations under the same criteria, both the height of the spillway and the chute slope angle were considered constant for all geometries.

Table 1: Characteristics of configurations of stepped spillway

Models	M1	M2	M3
Step Geometry L(m)	Flat	Quarter Cut	Half Cut
Step Height (m)	0.1	0.125	0.15
Step Length (m)	0.1	0.1	0.1
Number of step (N)	12	12	12
Slope	45º	45º	45º

In this paper, the meshed domain was defined as a 2-D model of flow over a stepped spillway. The meshes were created using the GMSH software as shown in Fig (1). The structured grid generation that is used in this study is highly recommended in the literature as well [26]. Moreover, mesh size has been determined carefully to obtain accurate results. The computation domain was discretized within a structured mesh with the size of 0.01 × 0.01 m^2 quadrilateral cells. For all models investigated, four boundary conditions were defined. The water inflow boundary was set as a velocity-inlet condition and obtained based on the range of flow rates $0.083 \leq Q \leq 0.216$ m^3/sec. The outlet boundary condition was characterized as an outlet pressure to permit the water to flow out freely. Additionally, the atmospheric pressure was considered at the free surface boundary, and Walls were assumed to have the no-slip condition.

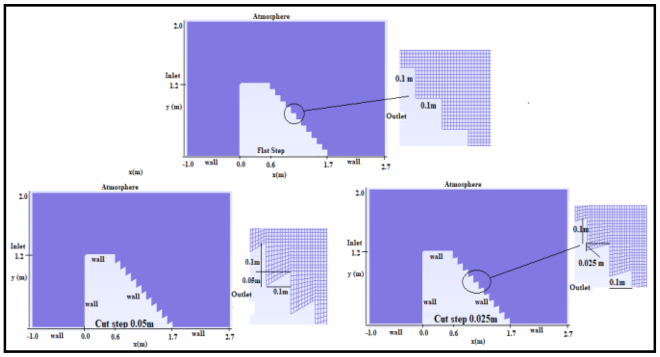


Fig. 1 Boundary conditions of structured mesh for stepped spillway configurations

2.1 Reynolds-Averaged Navier–Stokes (RANS)

The mathematical model considers the RANS equations are applied to isothermal, immiscible, incompressible flows describing the air –water, two –phase flow, problem [7, 27], by utilizing the traditional VOF (Volume of Fluid) tracking approach of a free surface, coupled with the turbulence closure models. The well-known Reynolds averaged Navier-Stokes formulas [28, 29] are the governing equations used in this work.

The Navier-Stokes formulas [(Eq. 1) and (Eq. 2)] can be composed as an equation for mass conservation and an equation for momentum conservation as:

$$\nabla . u = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u) + \nabla .(\rho u u) - \nabla .\left(\mu_{eff} \nabla u\right) = -\nabla p^{*} - g h \nabla \rho + F, \tag{2}$$

where, t is time, u is the mean component of the velocities, ρ is density, $p*$ is the modified pressure, g and h are gravity and position vectors, F is the volumetric surface tension force, $\mu_{eff} = (\mu + \mu_t)$, μ is the molecular viscosity, and μ_t is the term of turbulent viscosity, which can be determined by the turbulent model

2.2 The Volume of Fluid (VOF)

One of the main features of a stepped spillway is the air entrainment that causes two-phase flow. For analyses of two-phase flows in OpenFOAM, a VOF can be used and for a fixed Eulerian mesh, based on each fluid fraction, all the variables in every volume are computed. The advantage of the VOF is that a simple fluid model can be developed by a single equation [22, 7, 30]. To this end, the continuity equation for the volume portion of water is presented by Eq. (3):

$$\frac{\partial \alpha_w}{\partial t} + \nabla (u \alpha_w) = 0 \tag{3}$$

In the VOF mode, as the water and air phases share the same velocity and pressure field, the two-phase flow can be considered as a single-phase flow, where the turbulence model is homogeneous [7]. By solving Eq. (1), the free surface of water can be resolved. In all the computational cells, a fraction of water (αw) and air (αa) is considered such that: $\alpha_a = 1 - \alpha_w$. Therefore, the density (ρ) and molecular (μ) viscosity can be described by Eq. (2) and (3), respectively:

$$\rho = \alpha_w \rho_w + (1 - \alpha_w)\rho_a, \tag{4}$$

$$\mu = \alpha_w \mu_w + (1 - \alpha_w)\mu_a, \tag{5}$$

where, ρ_w and ρa are the density and μw and μa are the molecular viscosity of water and air, respectively. The maximum and minimum values of αw are 1 and 0, demonstrating that the given cell is filled up with water or air, respectively.

With the assistance of the Reynolds-averaged Navier–Stokes (RANS) equations [28], a two-phase flow is described in terms of a single pressure and velocity field that can be solved by the InterFOAM solver based on the VOF phase fraction.

2.3 Realizable k- ε Model (Rl k- ε).

In order to model the flow over a stepped chute by Eq. (1) and (2), a turbulence closure is required. The realizable k – ε model is a modification over the standard k− ε model that is given for a higher Reynolds number in the turbulence flow. The governing equations for this model; one for k, the other for ε are given through the following Eq. (6) and (7).

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+G_k+G_b-\rho\varepsilon-Y_M+S_K, \quad (6)$$

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\nabla(\rho\varepsilon u)=\nabla\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\nabla\varepsilon\right]+\rho C_1 S\varepsilon-\rho C_2\frac{\varepsilon^2}{k+\sqrt{v\varepsilon}}+C_{1\varepsilon}\frac{\varepsilon}{k}C_{3\varepsilon}G_b+S_\varepsilon, \quad (7)$$

$$C_1=max\left[0.43\frac{\eta}{\eta+5}\right],\eta=S\frac{k}{\varepsilon},S=(2S_{ij}S_{ij})^{0.5},$$

where: υ = kinematic viscosity, G_k is added because of the impact of the turbulence on the kinetic energy because of the mean velocity in the gradients, G_b is the generation of turbulent kinetic energy because of buoyancy, Y_M is used as the fluctuating expansion that is given in the turbulence. The equations incorporate some adjustable constants as: S_ε, S_k that are user-defined source terms, $C_{1\varepsilon}$ = 1.44, $C_{3\varepsilon}$ = 1.0, σ_k = 1, σ_ε = 1.2, and C_2 = 1.9.

2.4 Energy Dissipation Equation

For efficient structure, it is important to decide the quantity of energy dissipation in stepped spillways, $\Delta E/Et$, at the downstream end of the stepped spillway [11, 31], where ΔE is the total head loss ($\Delta E = Et - H_{res}$), Et is the maximum upstream head, and H_{res} is the residual energy calculated as:

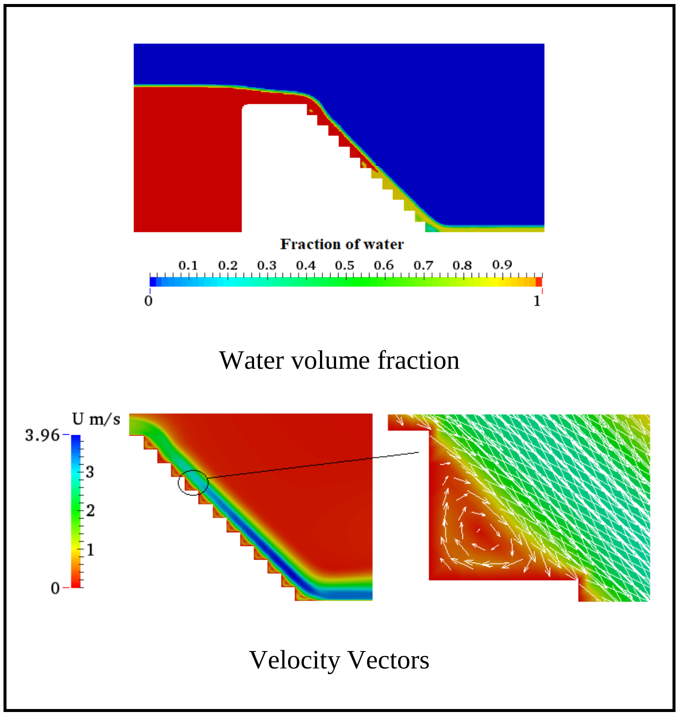
$$H_{res}=\int_{y=0}^{y90}(1-C)dy\times cos\theta+\frac{{q_w}^2}{2\times g\times\left(\int_{y=0}^{y90}(1-C)dy\right)^2}+Z, \quad (8)$$

where, C is the void fraction, y is measured perpendicular to the pseudo-bottom formed by the step edges, y_{90} is the depth where the local air concentration is 90% and Z is the step edge elevation above the datum.

3 RESULTS AND DISCUSSION

3.1 Numerical Validation

In this section, the validation of numerical results through experimental data is discussed. The validation consists of velocity profiles and energy dissipation. Experimental data obtained from Zhang and Chanson [32] were used to validate the numerical outcomes through the comparison of the velocity distribution, and energy dissipation. Fig (2) shows water fraction and velocity vectors distribution of flow on the stepped spillway. As it can be seen, the velocity vectors rotate clockwise above the horizontal surface of the step and the velocity increases from the center of eddies to the flow surface. Hence, the energy dissipation is carried by the recirculated flow in under pseudo-bottom.



Water volume fraction

Velocity Vectors

Fig. 2 Water volume fraction and velocity vectors on stepped chute at slope 45° for $Q = 0.145 m^3/sec$

Fig (3) shows a comparison of the velocity profiles between the laboratory measurements and the numerical simulation, where the measurement of the velocity was taken to the pseudo-bottom from the first step edge. Through the mean absolute percentage error (MAPE) represented by Eq. (9), the numerical simulations produced reliable and acceptable results.

$$MAPE=\frac{100}{n}\sum_{t=1}^{n}\left|\frac{A_t-F_t}{A_t}\right|, \quad (9)$$

where, n is the number of stations, A_t is the laboratory values, F_t is the forecast value.

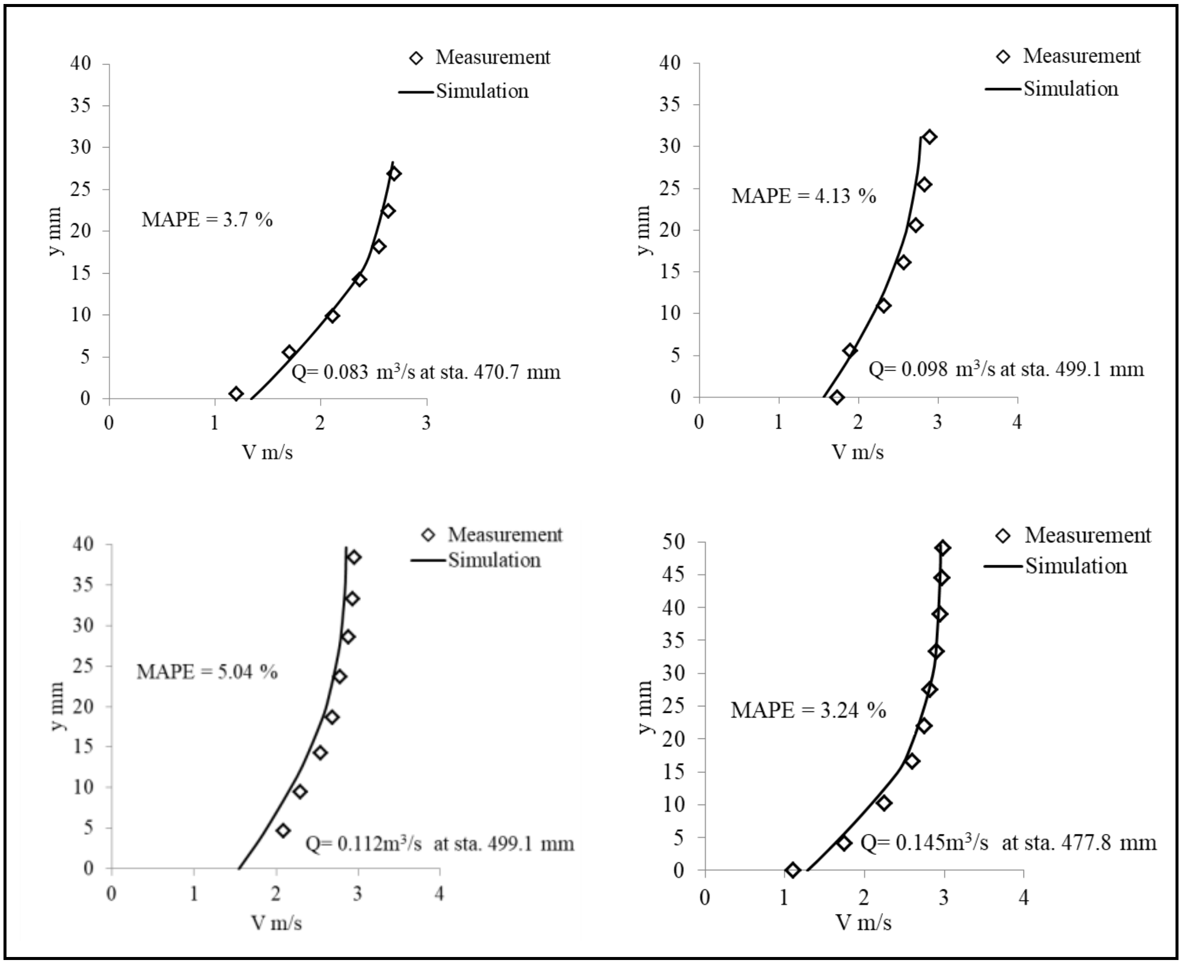


Fig. 3 Comparison of velocity distribution between numerical outcomes and experimental data in some sections.

For various discharges, within the range of $0.083 \leq Q \leq 0.216 m^3/sec$, the energy dissipation over a stepped chute was simulated. As it can be seen in Table 2, the numerically obtained outcomes are very similar to the experimental results reported by Zhang and Chanson [31], indicating a high degree of accuracy of the numerical experiments.

Table 2:The experimental and numerical energy dissipation results of the last step for flat steps

Reference	θ	H_{dam}	N	h (m)	L (m)	Q m^3/sec	d_c/h	ΔE/E_t		Error %
								Experiment Results	Numerical Results	
Zhang and Chanson [31]	45	1.2	12	0.1	0.1	0.057	0.7	0.61	0.622	1.967
						0.083	0.9	0.63	0.602	4.444
						0.098	1	0.63	0.605	3.968
						0.112	1.1	0.62	0.601	3.065
						0.145	1.3	0.60	0.572	4.667
						0.179	1.5	0.57	0.534	6.316
						0.216	1.7	0.53	0.501	5.472

3.2 Effect of cut steps on Energy Dissipation

The influence of steps cut on the rate of energy dissipation for all models is discussed in this section. Fig (4) shows the water volume fraction of flow, indicating recirculating flow in step cavities. Furthermore, there were continuous exchanges of air and water, and momentum between the mainstream and atmosphere while intense cavity recirculation was observed. Vortices were presented within the step cavities. These vortices are maintained through the transport of momentum by turbulent motions between the flow stream and the recirculating fluid underneath. As illustrated in Fig (4), the main cause of the energy dissipation on stepped spillways were the recirculation of the flow in step cavities.

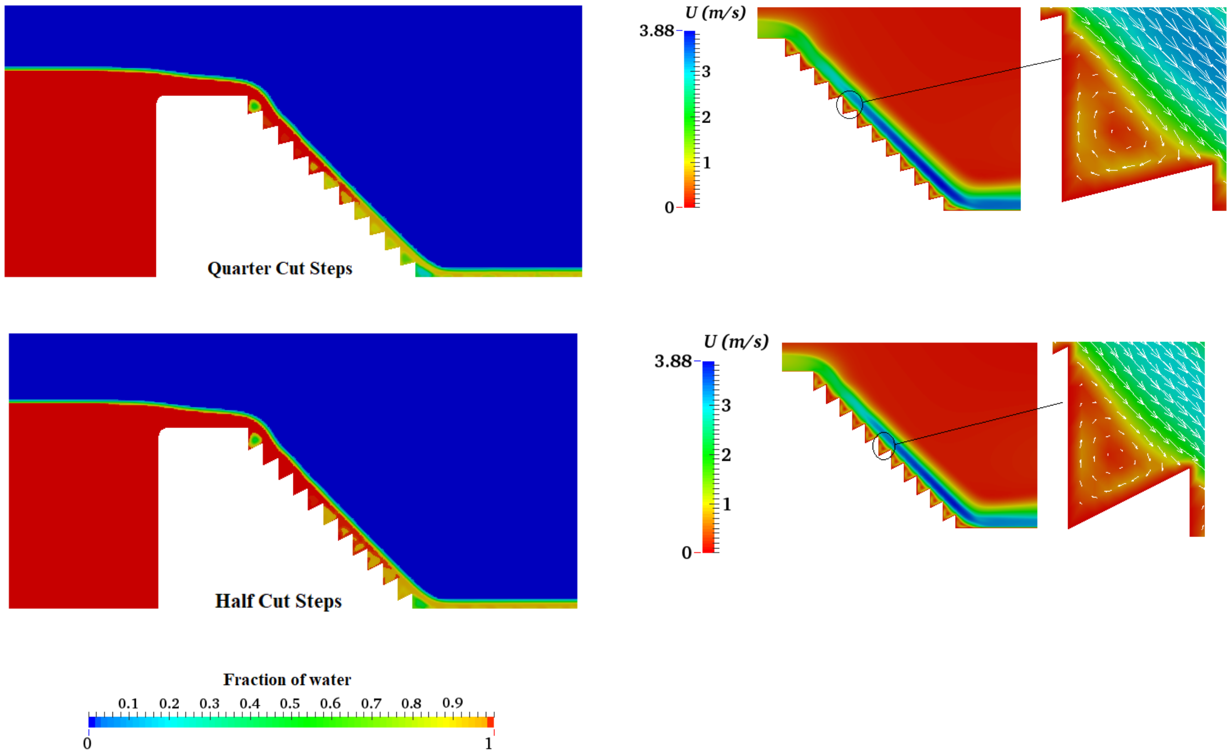


Fig. 4 Water volume fraction and velocity vectors on stepped chute with quarter and half cut steps for $Q = 0.145 m^3/sec$

Fig (5) shows the effect of the stepped spillway with steps cut on the energy dissipation performance. It can be seen that, the energy dissipation has an inverse relationship with the discharge. As the flow rate increases, the energy dissipation at the end of the chute decreases.

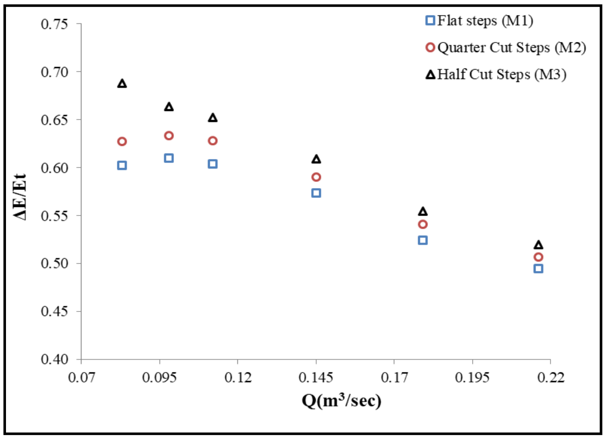


Fig. 5 Effect of steps cut on the rate of energy dissipation

The results show, half-cut steps (M3) have better dissipation than quarter-cut steps (M2); and, M2 has better dissipation than flat steps (M1). In which the total energy dissipation for M3 ranged from 52 to 69%, M2 ranged from 50 to 62%, and M1 ranged from 50 to 60%.

4 CONCLUSION

In this numerical work, an attempt to improve the energy dissipation using novel step shapes which was; flat step, quarter-cut steps, and half-cut steps. Based on the results obtained in this study, the main conclusions are:

1. The findings from this study are an indication that the VOF and Realizable k- ε model through InterFoam solver is a powerful tool to simulate the flow over the stepped spillway and helps in better understanding of the flow behavior for more accurate design of stepped spillway.
2. It was found that the main factor that affects energy dissipation is the vortexes formed by the recirculation flow. For all stepped chute models, the energy dissipation reduces when the flow rate increases.

3. The results indicated that that energy losses reduce with an increment of discharges as it enters a phase where the impact of the shape of the steps is exceptionally little at Q = 0.216m^3/sec.
4. Among the three configurations tested, the half-cut steps were the most effective, such that the energy dissipation was improved up to 8%.

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