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On β-Generalized Open and Closed Sets in Neutrosophic Topological Spaces

Rukaia M. Rashed

*Corresponding author: r.rasheed@zu.edu.ly
Department of Mathematics, Faculty of Education, Al-Zawia University, Libya)

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Abstract

This article demonstrates a class of neutrosophic closed sets named neutrosophic generalized βg −closed sets, discusses their essential characteristics in neutrosophic topological spaces, and analyses some new interesting theorems based on the newly introduced set. It also discusses its relationship between basic open and closed sets in neutrosophic topological spaces.

Keywords: Neutrosophic sets, neutrosophic topology, neutrosophic generalized βg −closed sets, and neutrosophic generalized βg −open sets.

INTRODUCTION

The concept of nutrosophic sets was first introduced by Floretin Smarandache (Floretin S.2010) in 1999, which is a generalization of intuitionistic fuzzy sets by Atanassov (Atanassov K. 1986). In (Dhavaseelan R. & Jafari S.2017), a generalized neutrosophic closed set (in short, $N_g CS$) is defined, and using this generalized neutrosophic continuous, generalized neutrosophic irresolute functions are defined.

Recently in (Dhavaseelan R., Jafari S. & Hani Md. 2018, Dhavaseelan R. & Hani Md. 2019), the perception of generalized α-contra continuous and neutrosophic almost α-contra-continuous functions are introduced.

In 1999, the neutrosophic sets and neutrosophic topological spaces by Salama A. A. and Alblowi S. A. were extended (Rena T. & Anila S.2018). Furthermore, the basic sets like neutronsophic open sets (NOS), neutrosophic semiopen sets (NSOS) neutrosophic pre-open sets (NPOS), neutrosophic α open sets ($N_\alpha OS$), neutrosophic regular open sets (N-ROS), neutrosophic β open sets ($N_\beta OS$), and neutrosophic b open sets (N-bOS) are introduced in neutrosophic topological spaces and their properties are studied by various authors (Pushpaiatha A.& Nandhini T.2019). This paper introduces the new concept of neutrosophic closed sets called generalized neutrosophic β closed and open sets and some of their basic properties with examples.

1- PRELIIMINARIES:

In the following section, we assume that (X, τ) is the neutrosophic topological space, let A be a neutrosophic set in X and it is an open set. Then we symbolize it by $NSO(A)$, and the complement of A is termed a neutrosophic closed set in X, also symbolized by $NSC(A)$. Also, the neutrosophic interior is denoted by $Nint(A)$, neutrosophic closure is denoted by $Ncl(A)$, and the empty band whole sets are denoted by 0 & 1 respectively.

Definition 1.1 (Abd El Monsef M.E. 1980)
A sub set A of topological space (X,τ) is called β −open (or semi pre open[6] if
$A \subset cl\left(int(cl(A))\right)$.

Definition 1.2 (Levine N. 1970)
A sub set A of topological space (X,τ) is called a generalized closed set ($g -$ closed for short) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is an open set. The complement of $g -$ closed set is called a g −open set.

Definition 1.3 (Dunham W. 1982)
If A is a subset of a space (X,τ), then
1- The generalized closure of A is defined as the intersection of all g −closed sets in X containing A and is denoted by $gcl(A)$ where$gcl(A) = \bigcap\{F: F \; is \; g - closed \; \& \; A \subseteq F\}$.
2- The generalized interior of A is defined as the union of all g −open sets in X contained in A and is denoted by $gint(A)$ where
$gint(A) = \bigcup\{G: G \; is \; g - open \; \& \; G \subseteq A\}$.
Definition 1.4 (Pushpaiatha A. & Nandhini T.2019)
For subset A of topological space (X,τ), then
1- The β −closure of A is the intersection of all $\beta -$ closed set that contain A. They are denoted by $\beta cl(A)$.
2- The $\beta -$ interior of A is the union of all $\beta -$ open sets contained in A. They are denoted by $\beta int(A)$.

Definition 1.5 (Salama A.A. & Alblowi S.A. 2012)
Let X be a non-empty fixed set, A neutrosophic set $NS - (A)$ is an object having the form $A = \{\langle X, \mu_A(x), \delta_A(x), V_A(x)\rangle : x \in X\}$ where $\mu A(x), \delta_A(x)$ & $V_A(x)$ represent the degree of membership, degree of indeterminacy, and the degree of nonmembership respectively of each element $x \in X$ to the set A. A
neutrosophic set
$A = \{\langle X, \mu_A(x), \delta_A(x), V_A(x)\rangle : x \in X\}$ can be identified as an ordered triple $\langle \mu_A, \delta_A, V_A \rangle$ in $]0,1[$ or X.

Definition 1.6 (Salama A.A. and Alblowi S.A. 2012)
Let $A = \langle \mu_A, \delta_A, V_A \rangle$ be a NS on X, then the complement $C(A)$ may be defined as
1-
$C(A) = \{\langle x, 1 - \mu_A(x), 1 - \delta_A(x), 1 - V_A(x)\rangle : x \in X\}$.

2- $C(A) = \{\langle x, V_A(x), \delta_A(x), \mu_A(x)\rangle : x \in X\}$.
3-
$C(A) = \{\langle x, V_A(x), 1 - \delta_A(x), \mu_A(x)\rangle : x \in X\}$.

Note that for any two neutrosophic sets A & B

4- $C(A \cup B) = C(A) \cap C(B)$.

5- $C(A \cap B) = C(A) \cup C(B)$.

Definition 1.7 (Salama A.A. & Alblowi S.A. 2012)

For any two neutrosophic sets $A = \{\langle x, \mu_A(x), \delta_A(x), V_A(x)\rangle : x \in X\}$, and $B = \{\langle x, \mu_B(x), \delta_B(x), V_B(x)\rangle : x \in X\}$ we may have

1- $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x)$, $\delta_A(x) \ge \delta_B(x)$, $V_A(x) \ge V_B(x), \forall x \in X$.

2- $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \delta_A(x) \vee \delta_B(x), V_A(x) \vee V_B(x)\rangle$

3- $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \delta_A(x) \wedge \delta_B(x), V_A(x) \wedge V_B(x)\rangle$

Definition 1.8 (Salama A.A. and Alblowi S.A. 2012)

A neutrosophic topology (in short, NT) on $X \neq \emptyset$ is a family τ of $N-$sets in X satisfying the laws given below

1- $0_N, 1_N \in \tau$.

2- $W_1 \cap W_2 \in \tau$ being $W_1, W_2 \in \tau$.

3- $\bigcup W_i \in \tau$ for the arbitrary family $\{W_i : i \in A\} \subseteq \tau$.

In this case the pair (X, τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (NOS) in X. A neutrosophic set A is a neutrosophic closed set (NCS) if and only if its complement $C(A)$ is a neutronsophic open set in X.

Definition 1.9

A neutrosophic A in a neutrosophic topological space (X, τ) is said to be

1- A neutrosophic β-open set $(N_\beta OS)$ if $A \subseteq Ncl\left(Nint\left(Ncl(A)\right)\right)$.

2- A neutrosophic β-closed set$(N_\beta CS)$ if $Nint\left(Ncl\left(Nint(A)\right)\right) \subseteq A$.

Remark 1.1

Note that $Ncl\left(C(A)\right) = C\left(Nint(A)\right)$ & $Nint\left(C(A)\right) = C\left(Ncl(A)\right)$.

Proposition 1.1 (Salama A.A. and Alblowi S.A. 2012)

Let (X, τ) be NTS and A, B be two neutrosophic sets in X, then the following properties hold:

(a) $Nint(A) \subseteq A$, (b) $A \subseteq Ncl(A)$,

(c) $A \subseteq B \Longrightarrow Nint(A) \subseteq Nint(B)$,

(d) $A \subseteq B \Longrightarrow Ncl(A) \subseteq Ncl(B)$,

(e) $Nint\left(Nint(A)\right) = Nint(A) \wedge Nint(B)$, (f) $Ncl(A \cup B) = Ncl(A) \vee Ncl(B)$,

(g) $Nint(1_N) = 1_N$, (h) $Ncl(0_N) = 0_N$.

Proposition 1.2 (Salama A.A. & Alblowi S.A. 2012)

For any neutrosophic set A in (X, τ) we have

(a) $Ncl\big(C(A)\big) = C\big(Nint(A)\big)$,

(b) $Nint\big(C(A)\big) = C\big(Ncl(A)\big)$.

Proposition 1.3 (Salama A.A. & Alblowi S.A. 2012),

For all, $A\ B$ two neutrosophic sets then the following are true

(a) $C(A \cap B) = C(A) \cup C(B)$,

(b) $C(A \cup B) = C(A) \cap C(B)$.

Definition 1.10 (Salama A.A.& Alblowi S.A. 2012)

Let A be an NS in an NTS (X, τ), there for

1- $Nint(A) = \bigcup\{G: G \text{ is a } NOS \text{ in } X \text{ and } G \subseteq A\}$ is termed as neutrosophic interior ($Nint$ for short) of A.

2- $Ncl(A) = \bigcap\{G: G \text{ is an } NCS \text{ in } X \text{ and } G \supseteq A\}$ is termed as neutrosophic closure (Ncl for short) of A.

Definition 1.11 (Pushpaiatha A. & Nandhini T. 2019)

$A\ NS\ \ A$ in $NTS\ X$ is so called a neutron sophic generalized closed set denoted by N_gCS if for any $NOS\ \ U$ in X such that $A \subseteq U$, then $Ncl(A) \subseteq U$. Moreover, its complement is named a neutrosophic generalized open set and referred to N_gOS.

Definition 1.12 (Dhavaseelan R. & Jafari S. 2017)

Let (X, τ) be NTS and B be a NS in X, then neutrosophic generalized closure is defined as $N_gcl(B) = \bigcap\{G: G \text{ is a } GNCS \text{ in } X \text{ and } B \subseteq G\}$.

$N_gint(B) = \bigcup\{U: U \text{ is a } GNOS \text{ in } X \text{ and } U \subseteq B\}$.

Proposition 1.4 (Salama A.A. & Alblowi S.A. 2012),

For any generalized neutrosophic set A the following are holds:

$0_N \subseteq A$, $0_N \subseteq 0_N$, $A \subseteq 1_N$, $1_N \subseteq 1_N$

Proposition 1.5 (Salama A.A. & Alblowi S.A. 2012)

Let (X, τ) be a $GNTS$ and A , B be two neutrosophic sets in X. Then the following properties hold:

(a) $Gint(A) \subseteq A$, (b) $A \subseteq GNcl(A)$, (c) $A \subseteq B \Longrightarrow GNint(A) \subseteq GNint(B)$,

(d) $A \subseteq B \Longrightarrow GNcl(A) \subseteq GNcl(B)$,

(e) $GNint\big((A \cap B)\big) = GNint(A) \wedge GNint(B)$,

(f) $GNcl\big((A\cup B)\big) = GNcl(A)\vee GNcl(B)$,
(g) $GNint(1_N) = 1_N$, $GNcl(0_N) = 0_N$.

Proposition 1.6 (Salama A.A.& Alblowi S.A. 2012)

For any generalized neutrosophic set A in (X,τ) we have
(a) $GNcl\big(C(A)\big) = C\big(GNint(A)\big)$,
(b) $GNint\big(C(A)\big) = C\big(GNcl(A)\big)$.

Definition 1.13

Let A be a neutrosophic set of a neutrosophic topological space (X,τ), then the neutrosophic β-interior and the neutrosophic β-closure are defined as
$N_\beta int(A) = \cup\{U: U\ isN\beta OS\ in\ X\ \&U \subseteq A\}$,
$N_\beta cl(A) = \cap\{F: F\ is\ N\beta CS\ in\ X\ \&\ A \subseteq F\}$.

Proposition 1.7

Let A be an a neutrosophic set in X, then
1- $N_\beta cl(A) = A \cup Nint\Big(Ncl\big(Nint(A)\big)\Big)$.
2- $N_\beta int(A) = A \cap Ncl\Big(Nint\big(Ncl(A)\big)\Big)$.

Proof:

1- We need to prove that $N_\beta cl(A) \subseteq A \cup Nint\Big(Ncl\big(Nint(A)\big)\Big)$ &
$A \cup Nint\Big(Ncl\big(Nint(A)\big)\Big) \subseteq N_\beta cl(A)$
Since $N_\beta cl(A) \subseteq N_\beta CS(A) \Longrightarrow$
$Nint\left(Ncl\left(Nint\left(N_\beta cl(A)\right)\right)\right) \subseteq N_\beta cl(A)$
$\Longrightarrow A \cup Nint\Big(Ncl\big(Nint(A)\big)\Big)$
$\subseteq A \cup Nint\left(Ncl\left(Nint\left(N_\beta cl(A)\right)\right)\right)$
$\subseteq A \cup N_\beta cl(A) = N_\beta cl(A). \qquad \longrightarrow (1)$
On the other hand, since we have
$Nint\Big(Ncl\big(Nint((A))\big)\Big) \cup$
$Nint\Big(Ncl\big(Nint((A))\big)\Big)$

$\subseteq Nint\left(Ncl\left(Nint\left(\left(A \cup Ncl(A)\right)\right)\right)\right)$
$= Nint\Big(Ncl\big(Nint(Ncl(A))\big)\Big)$
$= Nint\Big(Ncl\big(Nint((A))\big)\Big)$
$\subseteq A \cup Nint\Big(Ncl\big(Nint((A))\big)\Big). \qquad \longrightarrow (2)$
From (1) & (1) we get

$N_\beta cl(A) = A \cup Nint\left(Ncl\left(Nint(A)\right)\right)$.

2- The proof of this case similar to paragraph 1.

2- β- Generalized Closed and Open Sets in Neutrosophic Topological Spaces:

In this section we interduce concepts of the neutrosophicclosure, neutrosophic-β β-interior and β-generalized closed and open sets and its respective open set in neutrosophic topological spaces and discuss some of their properties.

Definition 2.1 (Rukaia M. Rashed 2020)

A sub set A of a topological space (X,τ) is called a β-generalized closed set $(g\beta - closed)$ if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \beta O(X)$.

Definition 2.2 (Rukaia M. Rashed 2020)

A subset A of a topological space (X,τ) is said to be a generalized β −open ($g\beta$ −open for short) set if $U \subseteq \beta int(A)$ where ever $U \subseteq A$ and U is closed. The complement of generalized β −open set is said to be generalized β −closed. The family of all $g\beta$ −open (resp. $g\beta$ −closed) sets of X is denoted by $G\beta O(X)$ (resp.$PG\beta C(X)$).

Proposition 2.1

Let (X,τ) be a neutrosophic topological space, then the union of any two $N_\beta OS$ in a $NTSX$ is a $N_\beta OS$.

Proof:

Let A & B be two $N_\beta OS$, therefore

$A \subseteq Ncl\left(Nint\left(Ncl(A)\right)\right)$ &

$B \subseteq Ncl\left(Nint\left(Ncl(B)\right)\right) \Longrightarrow$

$A \cup B \subseteq$

$Ncl\left(Nint\left(Ncl(A)\right)\right) \cup Ncl\left(Nint\left(Ncl(B)\right)\right)$

$= Ncl\left(Nint\left(Ncl(A)\right) \cup Nint\left(Ncl(B)\right)\right)$

$\subseteq Ncl\left(Nint\left(Ncl(A) \cup Ncl(B)\right)\right)$

$= Ncl\left(Nint\left(Ncl(A \cup B)\right)\right)$, then

$A \cup B \subseteq Ncl\left(Nint\left(Ncl(A \cup B)\right)\right)$,

so that $A \cup B$ is a $N_\beta OS$ in X.

Remark 2.1

The intersection of any two $N_\beta OS$ of an NTS does not have to be a $N_\beta OS$ as in this example.

Example 2.1

Let $X = \{a,b\}$ & $\tau = \{0_N, 1_N, A, B, M, N\}$ is a NTS on X where

$A = \langle x, (.3,.4), (.2,.1), (.7,.5)\rangle$,

$B = \langle x, (.2,.5), (.3,.4), (.4,.5)\rangle$,

$M = \langle x, (.3,.5), (.3,.4), (.4,.5)\rangle$,

$N = \langle x, (.2,.4), (.2,.1), (.7,.5)\rangle$, and let

$K_1 = \langle x, (.8, .4), (.1, .2), (.5, .7) \rangle$,
$K_2 = \langle x, (.5, .6), (.2, .5), (.3, .3) \rangle$.Then $Ncl\left(Nint\left(Ncl(K_1)\right)\right) = 1_N$, $Ncl\left(Nint\left(Ncl(K_2)\right)\right) = 1_N$, therefor K_1 & K_2 are $N_\beta OS$ in X. But $K_1 \cap K_2 = \langle x, (.5, .4), (.1, .2), (.5, .7) \rangle$ is not $N_\beta OS$ in X.

Proposition 2.2

Let A be any neutrosophic set in a neutrosophic topological space X, and let $A \subseteq B \subseteq Ncl(A)$, then B is a $N_\beta OS$ set in X.

Proof:

Since A is a $N_\beta OS$ set so that
$A \subseteq Ncl\left(Nint\left(Ncl(A)\right)\right) \Longrightarrow$
$Ncl(A) \subseteq Ncl\left(Ncl\left(Nint\left(Ncl(A)\right)\right)\right)$
$= Ncl\left(Nint\left(Ncl(A)\right)\right) \Longrightarrow$
$Ncl(A) \subseteq Ncl\left(Nint\left(Ncl(A)\right)\right)$, since
$A \subseteq B \subseteq Ncl(A)$, then
$B \subseteq Ncl\left(Nint\left(Ncl(A)\right)\right)$. Also $A \subseteq B \Longrightarrow$
$Ncl\left(Nint\left(Ncl(A)\right)\right) \subseteq Ncl\left(Nint\left(Ncl(B)\right)\right)$,
hence $B \subseteq Ncl\left(Nint\left(Ncl(B)\right)\right)$, then B is a $N_\beta OS$ set in X.

Proposition 2.3

Let(X, τ) be a neutrosophic topological space, and A be a neutrosophic set of X. Then A is $N_\beta CS$ if and only if $C(A)$ is a $N_\beta OS$.

Proof:

Suppose that A is a $N_\beta CS$ in X. Then $Nint\left(Ncl\left(Nint(A)\right)\right) \subseteq A$, taking the compliment of both sides, then we have

$C(A) \subseteq C\left(Nint\left(Ncl\left(Nint(A)\right)\right)\right) =$
$Ncl\left(Nint\left(Ncl\left(C(A)\right)\right)\right) \Longrightarrow$

$C(A) \subseteq Ncl\left(Nint\left(Ncl\left(C(A)\right)\right)\right)$, therefore
$C(A)$ is $N_\beta OS$ in X.
On the other hand, suppose that $C(A)$ is $N_\beta OS$ in X. So that
$C(A) \subseteq Ncl\left(Nint\left(Ncl\left(C(A)\right)\right)\right)$, taking the complement of both sides we have

$C\left(Ncl\left(Nint\left(Ncl(C(A))\right)\right)\right) \subseteq A \Longrightarrow$
$Nint\left(Ncl(Nint(A))\right) \subseteq A.$
Then A is a
$N_\beta CS$ in X.

Proposition 2.4

The intersection of any two $N_\beta CS$ of an NTS, is also $N_\beta CS$.

Proof

Suppose that A & B are two $N_\beta CS$ in X. So $Nint\left(Ncl(Nint(A))\right) \subseteq A$ & $Nint\left(Ncl(Nint(B))\right) \subseteq B$, then
$Nint\left(Ncl(Nint(A))\right) \cap$
$Nint\left(Ncl(Nint(B))\right) \subseteq A \cap B \Longrightarrow$

$Nint\left(Ncl(Nint(A \cap B))\right) \subseteq A \cap B$. Then
$A \cap B$ is a $N_\beta CS$.

Remark 2.2

Note that the union of any two $N_\beta CS$ in X is not a $N_\beta CS$ as in the following example:

Example 2.2

Let $X = \{a\}$ & $\tau = \{0_N, 1_N, A, B\}$ be a NTS on X where $A = \langle x, (.2), (.5), (.3)\rangle$,
$B = \langle x, (.1), (.5), (.7)\rangle$, and let
$K_1 = \langle x, (.0), (.5), (.8)\rangle$,
$K_2 = \langle x, (.1), (.2), (.3)\rangle$. Then
$Nint(K_1) = 0_N$ & $Nint(K_2) = 0_N$.
Therefore, K_1,
K_2 are $N_\beta CS$, but $K_1 \cup K_2$ is not $N_\beta CS$.

Proposition 2.5

Every NCS in X is a $N_\beta CS$.

Remark 2.3

The converse of the above proposition is not true in the general, as in the following example:

Example 2.3

Let $X = \{a, b, c\}$ & $\tau = \{0_N, 1_N, A, B\}$ is a NTS on X where
$A = \langle x, (.5, .1, .1), (.6, .7, .6), (.3, .9, .4)\rangle$,
$B = \langle x, (.0, .1, .5), (.4, .6, .5), (.7, .9, .8)\rangle$,
and let

$K = \langle x, (.2, 0, .3), (.4, .2, .2), (.9, .9, .1) \rangle$.
Then K is a $N_\beta CS$ but not a NCS.

Proposition 2.6

Let A be a $N_\beta CS$, and $Nint(A) \subseteq B \subseteq A$, then B is a $N_\beta CS$.

Proof

Suppose that A is a $N_\beta CS$, so
$Nint\left(Ncl\left(Nint(A)\right)\right) \subseteq A$, then we have
so $Nint\left(Ncl\left(Nint(A)\right)\right) \subseteq Nint(A)$, and we have $Nint(A) \subseteq B$. Then, it follows that $Nint\left(Ncl\left(Nint(A)\right)\right) \subseteq B$, and $B \subseteq A \Longrightarrow$
$Nint\left(Ncl\left(Nint(B)\right)\right) \subseteq$
$Nint\left(Ncl\left(Nint(A)\right)\right)$,

so $Nint\left(Ncl\left(Nint(B)\right)\right) \subseteq B$, then B is a
$N_\beta CS$.

Proposition 2.7

For any $NS\ A$ in $TS\ \tau$, the subsequent features stand:

1- $N_{\beta_g}int(\overline{A}) = N_{\beta_g}int(A)$.

2- $N_{\beta_g}cl(\overline{A}) = N_{\beta_g}cl(A)$.

Proof

The proof will be evident by symbolic definition,

$Neg\beta cl(A) = \bigcap\{F: A \subseteq F, F \text{ is a } Ne - g\beta CS\}$

1- $Ne - g\beta cl(A) = \bigcap\left\{\overline{F}: \overline{A} \subseteq \overline{F}, \overline{F} \text{ is a } Ne - g\beta C(S)\right\}$

$= \bigcup\{U: A \supseteq U, U \text{ is a } Ne - g\beta OS\}$

$= Ne - g\beta int(\overline{A})$.

2- This feature has undeniable proof analogous to feature (1).

Proposition 2.8

For any $N_{\beta_g}OS\ A$ in $TS\ \tau$, then this set is $N_\beta OS$ (corresponding $N_{\beta_g}OS$).

Proof

Similar to the proof of the previous theorem.

Definition 2.3

A neutrosophic A in a neutrosophic topological space X is said to be a neutrosophic β-generalized closed set $(N_{\beta_g}CS)$ if
$N_\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a
NOS in X. The complement $C(A)$ of a
$N_{\beta_g}CSA$ is a $N_{\beta_g}OS$ in X.

Definition 2.4

A neutrosophic A in a neutrosophic topological space X is said to be a neutrosophic β-generalized open set $(N_{\beta_g}OS)$ if
$U \subseteq N_\beta int(A)$ whenever $U \subseteq A$ and U is a N-closed set.

Example 2.4

Let $X = \{a, b\}$ and $\tau = \{0_N, 1_N, A, B\}$
where $A = \langle x, (.5, .6), (.3, .2), (.4, .1)\rangle$ &
$B = \langle x, (.4, .4), (.4, .3), (.5, .4)\rangle$ then τ is a neutrosophic topology. Hence let
$M = \langle x, (.5, .4), (.4, .4), (.4, .5)\rangle$ be any NS
in X then $M \subseteq A$ where A is a NOS in X. Now $N_\beta cl(M) =$
$\cap\{F: F\ is N_\beta CS\ in\ X\ \&\ M \subseteq F\} = C(B) \subseteq A$, or $N_\beta cl(M) = M \cup C(B) = C(B) \subseteq A$.
Therefore M is a $N_{\beta_g}CS$ in X.
$C(A) = \langle x, (.4, .1), (.3, .2), (.5, .6)\rangle$,
$C(B) = \langle x, (.5, .4), (.4, .3), (.4, .4)\rangle$, so A is
$N_\beta CS$ if $Nint\left(Ncl(Nint(A))\right) \subseteq A$
$\Rightarrow Nint\left(Ncl(Nint(A))\right) = o_N \subseteq A$,
also $Nint\left(Ncl(Nint(B))\right) = B \subseteq B$
$\Rightarrow A\ \&\ B$ are neutrosophic β-closed sets.
Now, $N_\beta cl(M) = A \nsubseteq U = C(B)$, so that M is not $N_{\beta_g}CS$.

Example 2.5

Let $X = \{a, b\}, \tau = \{0_N, 1_N, A, B\}$ is an
$NTS, A = \langle x, (.5, .3), (.5, .7), (.5, .7)\rangle$&
$B = \langle x, (.4, .3), (.6, .7), (.6, .7)\rangle$ are NS in X, if $M = \langle x, (.4, .6), (.4, .4), (.4, .4)\rangle$, then M is $N_{\beta_g}CS$ but does not $N_\beta CS$ in X, since
$Nint\left(Ncl(Nint(M))\right) = A \nsubseteq M$.

Proposition 2.9

Every $NCS\ A$ is a $N_{\beta_g}CS$ in X but not conversely in general.

Proof

Let $A \subseteq U$ where U is a NOS in X,
Now $N_\beta cl(A) = \left(N_\beta O(A)\right)^C =$
$\left(A \subseteq Ncl\left(Nint(Ncl(A))\right)\right)^C =$
$\left(A \subseteq Ncl(A)\right)^C = \left(A \cup Ncl(A)\right)^C =$
$(A \cup A)^C = A^C \subseteq U^C \Rightarrow A \subseteq U$,
by hypothesis therefore A is $N_{\beta_g} - CS$.

Proposition 2.10

Every $N_\beta CS\ A$ is a $N_{\beta_g}CS$ in X but the converse is not true in general.

Proof:

Let $A \subseteq U$, where A is$N_\beta CS, U$ is a $N_\beta OS$ in X, then $N_\beta C(A) = \left(N_\beta O(A)\right)^c$
$= \left(A \subseteq Ncl\left(Nint(Ncl(A))\right)\right)^c$
$= N_\beta C(A) = A \subseteq U$ (by previous *Proposition)*. Then we have $N_\beta C(A) \subseteq U$, hence A is an $N_{\beta_g}CS$ in X.

Proposition 2.11

Every $NOS, N_\beta OS$ are $N_{\beta_g}OS$ but not conversely in general.

Proof*:*

Obvious.

Remarks 2.4

1- The union of any two $N_{\beta_g}CS$ in a $NTSX$ is not a $N_{\beta_g}CS$ in a general case.

2- The intersection of any two $N_{\beta_g}CS$ need not be a $N_{\beta_g}CS$ in a $NTS\ X$ in general.

Example 2.7

Let $X = \{a, b\}$, and $\tau = \{0_N, 1_N, A, B, M\}$ be a neutrosophic topological space on X where $A = \langle x, (.5, .6), (.5, .4), (.5, .4)\rangle$ and $B = \langle x, (.2, .3), (.8, .7), (.8, .7)\rangle$, $M = \langle x, (.6, .7), (.4, .3), (.4, .3)\rangle$. Let $L_1 = \langle x, (.1, .5), (.9, .5), (.9, .5)\rangle$, $L_2 = \langle x, (.5, .2), (.5, .8), (.5, .8)\rangle$, then L_1 & L_2 are $N_{\beta_g}CS$ in X but $L_1 \cup L_2$ is not an $N_{\beta_g}CS$ since $L_1 \cup L_2 = \langle x, (.5, .5), (.5, .5), (.5, .5)\rangle \subseteq A$ but $N_\beta cl(L_1 \cup L_2) = \langle x, (.6, .7), (.4, .3), (.4, .3)\rangle \nsubseteq A$. Then M & N are $N_{\beta_g}CS_S$ in X but $M \cup N \subseteq B$ and $N_\beta cl(M \cup N) = 1_N \nsubseteq A$.

Example 2.8

Let $X = \{a, b\}$, and $\tau = \{0_N, 1_N, A, B, M\}$ is a neutrosophic topological space on X where $A = \langle x, (.5, .6), (.5, .4), (.5, .4)\rangle$ and $B = \langle x, (.2, .3), (.8, .7), (.8, .7)\rangle$, $M = \langle x, (.6, .7), (.4, .3), (.4, .3)\rangle$. Let $L_1 = \langle x, (.5, .8), (.5, .2), (.5, .2)\rangle$, $L_2 = \langle x, (.8, .6), (.2, .4), (.2, .4)\rangle$, then L_1 & L_2 are $N_{\beta_g}CS$ in X but $L_1 \cap L_2$ is not an $N_{\beta_g}CS$ since $L_1 \cap L_2 = \langle x, (.5, .6), (.5, .4), (.5, .4)\rangle \subseteq A$. But $N_\beta cl(L_1 \cap L_2) = \langle x, (.6, .7), (.4, .3), (.4, .3)\rangle \nsubseteq A$. Then M & N are $N_{\beta_g}CS_S$ in X but

$M \cap N \subseteq B$ and $N_\beta cl(M \cap N) = 1_N \not\subset A$.

Proposition 2.12

Let (X, τ) be a NTS. Then for every $A \in N_{\beta_g}C(X)$ and for every $B \in NS(X)$, $A \subseteq B \subseteq N_\beta cl(A)$ implies that $B \in N_{\beta_g}C(X)$.

Proof:

Let $B \subseteq U$ and U be a NOS in (X, τ). Then, since $A \in N_{\beta_g}C(X), \Longrightarrow$ $N_\beta cl(A) \subseteq U, A \subseteq U$ then, since
$B \subseteq N_\beta cl(A) \Longrightarrow$
$N_\beta cl(B) \subseteq N_\beta cl\left(N_\beta cl(A)\right) = N_\beta cl(A)$, so
$N_\beta cl(B) \subseteq N_\beta cl(A) \subseteq U$, then
$N_\beta cl(B) \subseteq U$, Hence $B \in N_{\beta_g}C(X)$.

Example 2.9

Let $X = \{a, b\}, \tau = \{0_N, 1_N, A, B\}$, such that $A = \langle X, (.5,.5), (.5,.5), (.5,.5)\rangle$
$B = \langle X, (.4,.3), (.6,.7), (.6,.7)\rangle$,
$S = \langle X, (.3,.2), (.7,.8), (.7,.8)\rangle \Longrightarrow S$ is
$N_\beta CS$ in X, why? We have
$C(A) = \langle X, (.5,.5), (.5,.5), (.5,.5)\rangle$,
$C(B) = \langle X, (.6,.7), (.6,.7), (.4,.3)\rangle$,
$C(S) = \langle X, (.7,.8), (.7,.8), (.3,.2)\rangle$ note that
$C(S)S$ is not NCS, since $C(S) \notin \tau$.
The $N_\beta OS$ are $A, B, C(A)$& $C(B)$ since
$A \subseteq Ncl\left(Nint\left(Ncl(A)\right)\right) = C(A)$,
$B \subseteq Ncl\left(Nint\left(Ncl(B)\right)\right) =$
$\langle X, (.5,.5), (.6,.7), (.5,.5)\rangle$,

$C(A) \subseteq Ncl\left(Nint\left(Ncl(C(A))\right)\right) = C(A)$ &
$C(B) \subseteq Ncl\left(Nint\left(Ncl(C(B))\right)\right) = B$.
The $N_\beta CS$ are $A, B, C(A)$ & $C(B)$, so that S is $N_\beta gCS$ if $N_\beta cl(S) \subseteq U, U$ is $N_\beta OS$
$N_\beta cl(S) = \cap\{K: K \text{ is a } N_\beta CS \text{ in } X \;\&S \subseteq$
$K\} = A \cap B \cap C(A) \cap C(B) = B \Longrightarrow S$
is
$N_\beta gCS$.

Proposition 2.13

If A is a NOS and a $N_{\beta_g}CS$ in (X, τ), then A is a $N_{\beta_g}CS$ in (X, τ).

Proof

Since $A \subseteq A$ and A is a NOS in(X,τ), by hypothesis, $N_\beta cl(A) \subseteq A$. But $A \subseteq N_\beta cl(A)$. There $N_\beta cl(A) = A$. Hence A is a $N_\beta CS$ in (X,τ).

Proposition 2.14

Every neutrosophic closed set in neutrosophic topological space (X,τ) is a neutrosophic generalized β −closed set.

Proof

Let A be a neutrosophic closed set in neutrosophic topological space X, let $A \subseteq U$ be a neutrosophic open set in X. Then by definition and previous proposition, we get $A = Ncl(A)$, $N_S cl(A) \subseteq Ncl(A)$, we get $N_S cl(A) \subseteq Ncl(A) = A \subseteq U$. Hence A is a neutrosophic generalized semi-closed set in X.

DISCUSSION

The results should be discussed in relation to any hypotheses advanced in the Introduction. Comment on results and indicate possible sources of error. Place the study in the context of other work reported in the literature. Only in exceptional cases should the "Results and Discussion" sections be combined. Refer to graphs, tables and figures by number (for example Figure 5 or Table 5. This helps tie the data into the text in a very effective manner.

CONCLUSION

This paper introduced and studied the notion of β −open and β −closed sets in a neutrosophic topology, and some characterizations of these notions are discussed.

In future research, we will extend these neutrosophic topology concepts by neutrosophic generalized β −continuous and neutrosophic β −generalized continuous in neutrosophic topological spaces. Also, we extend this neutrosophic concept by nets, filters, and neutrosophic $\beta\ g$ −compactnes.

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