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Application of the Sawi Transform to Chemical Differential Equation Models

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Abstract

In order to solve mathematical models in chemical sciences, this study compares the performance of the Sawi transform with the traditional Laplace transform. A first-order differential equation for reaction-diffusion processes, a model with time-dependent coefficients to evaluate flexibility, and a second-order equation describing single-step reversible reactions are the three different scenarios to which the Sawi transform is applied. Our findings show that the Sawi transform preserves exact analytical consistency with the Laplace transform while successfully reducing algebraic difficulty in the transformation process. In particular, the study showed that the Sawi transform produces identical exact solutions to standard methods while handling variable coefficients with improved procedural efficiency. These results demonstrate that the Sawi transform is a reliable and mathematically straightforward substitute for simulating complex chemical processes.

Keywords: Chemical Mixture; Chemical sciences; Inverse transform; Mathematical Modelling; Sawi transform.

INTRODUCTION

Mathematics plays a fundamental role in the formulation, analysis, and simplification of complex problems across a wide range of scientific, technical, economic, and artificial intelligence domains. In particular, differential equations constitute a central component of the mathematical framework used in physics, chemistry, biology, and economics, as they describe the dynamic behavior of evolving systems (Oldham et al., 1974; Zill, 2013; Gupta et al., 2022; Hilmi et al., 2024a; Faraj et al., 2023).

Integral transforms are commonly employed as analytical tools for solving differential equations by converting them into algebraic equations in a transformed domain, which are often more tractable. Among the most widely used transforms is the Laplace transform, which has been extensively applied to initial and boundary value problems. In this context, the Sawi transform can be viewed as a reformulation related to the Laplace transform through a transformation of the time variable, and it provides an alternative representation for handling such problems (Turab et al., 2024; Eshtewi, 2025; Hilmi et al., 2024b; Kamal Sedeeg, 2016; Patil et al., 2023a; Patil et al., 2023b). To illustrate, the temporal evolution of a quantity in a mixing process, subject to inflow and outflow rates, can be modeled by the first-order differential equation (Zill, 2013):
Integral transforms are also utilized in modeling more complex systems, including population dynamics and higher-order or fractional differential equations (Hilmi Jwamer et al., 2022). Such mod-

eling approaches have broad applications in advanced areas such as fractional calculus, physics, chemistry, and mechanics (Ayata et al., 2021; Jwamer et al., 2022).

The objective of this study is to investigate the application of the Sawi transform to differential equation models arising in mixing processes and related dynamical systems, and to provide a direct comparison with the Laplace transform (Schiff, 1999). The comparison focuses on the equivalence of the obtained solutions and their analytical representations. In addition, computational implementations using MATLAB are employed to generate illustrative plots for selected examples.

Preliminaries:

Basic properties of the Sawi transform.

Given a piecewise continuous function of exponential order, $f(t)$, defined in the interval, $[0,\infty)$, the Sawi transform of this function is given by

Definition 5: (Mahgoub, 2019) The Sawi transform of a piecewise continuous function of exponential order, $f(t)$ defined in the interval, $[0,\infty)$ is given by

$$S\{f(t)\} = \left(\frac{1}{v^2}\right)\int_0^\infty f(t)e^{-\frac{t}{v}}\,dt = F(v), \qquad t > 0\ .$$

Remark 1: The Sawi transform has a duality relation with the famous and widely used integral transform "Laplace transform", if $L\big(f(t)\big) = \int_0^\infty f(t)e^{-st}dt = F(s)$, where L is the Laplace transform, and $S\big(z(t)\big) = Z(v)$, then $Z(v) = \frac{F(\frac{1}{v})}{v^2}$.

Property 1: (Mahgoub, 2019) Some fundamental functions and their Sawi transforms are shown in Table (1).

Table (1) illustrates the Sawi transforms of some fundamental functions.

$f(t), t > 0$	$S\{f(t)\} = F(v)$	$f(t), t > 0$	$S\{f(t)\} = F(v)$
1	$\frac{1}{v}$	e^{kt}	$\frac{1}{v(1-kv)}$
t	1	$sin\,kt$	$\frac{k}{1+k^2v^2}$
t^2	$2!\,v$	$cos\,kt$	$\frac{1}{v(1+k^2v^2)}$
$t^n, n \in \mathbb{N}$	$n!\,v^{n-1}$	$sinh\,kt$	$\frac{k}{(1-k^2v^2)}$
$t^\beta, \beta > -1, \beta \in R$	$v^{\beta-1}\Gamma(\beta+1)$	$cosh\,kt$	$\frac{1}{v(1-k^2v^2)}$

Property 2: (Mahgoub, 2019) Inverse Sawi transforms for some fundamental functions are shown in Table (2).

Table (2) illustrates the inverse Sawi transforms of some fundamental functions.

$S^{-1}\{F(v)\} = f(t)$	$f(t), t > 0$	$S^{-1}\{F(v)\} = f(t)$	$f(t), t > 0$
$\frac{1}{v}$	1	$\frac{1}{v(1-kv)}$	e^{kt}
1	t	$\frac{k}{1+k^2v^2}$	$\sin kt$
$2!\,v$	t^2	$\frac{1}{v(1+k^2v^2)}$	$\cos kt$
$n!\,v^{n-1}$	$t^n, n \in \mathbb{N}$	$\frac{k}{(1-k^2v^2)}$	$\sinh kt$
$v^{\beta-1}\Gamma(\beta+1)$	$t^{\beta}, \beta > -1, \beta \in R$	$\frac{1}{v(1-k^2v^2)}$	$\cosh kt$

Property 3 (Patil et al., 2022) **(Convolution property):** If $S\{f(t)\} = F(v)$ and $S\{g(t)\} = G(v)$ then
$S\{f(t) * g(t)\} = v^2 F(v)G(v)$,
where * denotes the convolution of f and g, then $f(t) * g(t) = \int_0^t f(t-u)g(u)du$.

Property 4: Before obtaining the Sawi transform for $tf(t)$, $t^2f(t)$, the following steps are considered.

By differentiating the Sawi transform $S\{f(t)\} = v^{-2}\int_0^{\infty} f(t)e^{-\frac{t}{v}}dt = F(v)$ with respect to v, we obtain:

$$\frac{d}{dv}S\{f(t)\} = -2v^{-3}\int_0^{\infty} f(t)e^{-\frac{t}{v}}dt + v^{-4}\int_0^{\infty} t\,f(t)e^{-\frac{t}{v}}dt \qquad (1)$$

$$\Rightarrow \quad \frac{d}{dv}S\{f(t)\} = -\frac{2}{v}S\{f(t)\} + \frac{1}{v^2}S\{tf(t)\} \qquad ,$$

$$\Rightarrow \quad S\{tf(t)\} = v^2\frac{d}{dv}S\{f(t)\} + 2vS\{f(t)\} \qquad (2)$$

Differentiating Equation (1) with respect to v yields:

$$\frac{d^2}{dv^2}S\{f(t)\} = 6v^{-4}\int_0^{\infty} f(t)e^{-\frac{t}{v}}dt - 6v^{-5}\int_0^{\infty} t\,f(t)e^{-\frac{t}{v}}dt + v^{-6}\int_0^{\infty} t^2 f(t)e^{-\frac{t}{v}}dt$$

$$\Rightarrow \quad S\{t^2f(t)\} = v^4\frac{d^2}{dv^2}S\{f(t)\} - 6v^2S\{f(t)\} + 6vS\{tf(t)\}$$

By substituting $S\{tf(t)\}$ from Equation (2) into the preceding equation, it follows that:

$$S\{t^2f(t)\} = v^4\frac{d^2}{dv^2}S\{f(t)\} + 6v^3\frac{d}{dv}S\{f(t)\} + 6v^2S\{f(t)\} \qquad (3)$$

Property 5: (Hilmi, Mohammed Faeq, et al., 2024) The Sawi transform for integer-order derivative is given by

$$S\{f^{(m)}(t)\} = \frac{1}{v^m}F(v) - \sum_{k=0}^{m-1}\frac{1}{v^{m-k+1}}f^{(k)}(0).$$

Property 6: To calculate the Sawi transform of $tf'(t), tf''(t)$, we proceed as follows:

1- Substituting $f(t)$ into Equation (2) with $f'(t)$, we obtain:

$$S\{tf'(t)\} = v^2\frac{d}{dv}S\{f'(t)\} + 2vS\{f'(t)\}$$

Using Property (5), $S\{f'(t)\} = \frac{1}{v}F(v) - \frac{1}{v^2}f(0)$; accordingly, the previous equation can be written in the form:

$$S\{tf'(t)\} = -S\{f(t)\} + v\frac{d}{dv}S\{f(t)\} + \frac{2}{v}f(0) + 2S\{f(t)\} - \frac{2}{v}f(0),$$

Simplifying gives

$$S\{tf'(t)\} = v\frac{d}{dv}S\{f(t)\} + S\{f(t)\}$$

2- Substituting $f(t)$ into Equation (2) with $f''(t)$, we obtain:

$$S\{tf''(t)\} = v^2\frac{d}{dv}S\{f''(t)\} + 2vS\{f''(t)\}$$

Using Property (5), we have $S\{f''(t)\} = \frac{1}{v^2}F(v) - \frac{1}{v^3}f(0) - \frac{1}{v^2}f'(0)$; accordingly, the previous equation can be written in the form:

$$S\{tf''(t)\} = \frac{v^2 d}{dv}\left(\frac{1}{v^2}F(v) - \frac{1}{v^3}f(0) - \frac{1}{v^2}f'(0)\right) + 2v\left(\frac{1}{v^2}\,F(v) - \frac{1}{v^3}f(0) - \frac{1}{v^2}f'(0)\right),$$

By rearranging and collecting terms in the previous equation, we obtain

$$S\{tf''(t)\} \;= \frac{d}{dv}F(v) + \frac{1}{v^2}\,f(0)$$

Applications in Chemistry

In this section, we explore the application of the Sawi transform to various phenomena in chemical sciences. The Sawi transform provides an effective tool for solving mathematical models in this field.

Mixing problems are of particular importance in chemical sciencess. Consider a common example: a fixed-capacity tank containing a thoroughly mixed solution, such as salt in water. A solution of a specified concentration enters the tank at a fixed rate, and after thorough mixing, the resulting mixture exits the tank at a potentially different fixed rate.

Let $y(t)$ represent the quantity of material in the tank at time t. The rate of change of this quantity, $y'(t)$, is equal to the rate at which material enters the tank minus the rate at which it leaves. Such systems can often be modeled by a first-order differential equation, providing a clear mathematical framework for analyzing mixing phenomena.

The model is

$\frac{dy}{dt} = Rate\ of\ amount$, or $\frac{dy}{dt} = Rate\ in - Rate\ out$, with initial conditions

Problem 1: 30 kg of salt has been dissolved in 6000 liters of water in a tank. At a rate of 25 liters per minute, brine—which has 0.04 kilogram of salt per liter of water—enters the tank. The mixture is kept thoroughly blended, and the tank empties at the same pace. When 1 hour has passed, how much salt is still in the tank?

Solution: At the initial time $t\ = 0, y(t)$ denotes the amount of salt. Therefore, the tank contains 30 kg of salt i.e., $y(0) = 30$ and the amount of salt remaining after 60 min i.e., $y(60)$

$\frac{dy}{dt} = Rate\ of\ amount\ of\ salt$, or $\frac{dy}{dt} = Rate\ in - Rate\ out$

$Rate\ in = 0.04\ \left(\frac{kg}{L}\right) \times 25\ \left(\frac{L}{min}\right) = 1\ \left(\frac{kg}{min}\right),$

$Rate\ out = \left(\frac{y(t)}{6000}\right)\ \left(\frac{kg}{L}\right) \times 25\ \left(\frac{L}{min}\right) = \frac{y(t)}{240}\ \left(\frac{kg}{min}\right),$

From the model, we have $\frac{dy}{dt} = Rate\ in - Rate\ out \Rightarrow \frac{dy}{dt} = 1\ \left(\frac{kg}{min}\right) - \frac{y(t)}{240}\ \left(\frac{kg}{min}\right)$

$y'(t) + \frac{y(t)}{240} = 1$ we can use the Sawi transform to solve the first-order differential equation. The Sawi transform $S\{y(t)\}$ of a function $y(t)$ is defined as:

$$S\{y(t)\} = \left(\frac{1}{v^2}\right)\int_0^\infty y(t)e^{-\left(\frac{1}{v}\right)t}dt = Y(v), \qquad v > 0$$

Transform the differential equation: Applying the Sawi transform to both sides of the equation:

$$S\left\{y'(t) + \frac{y(t)}{240}\right\} = S\{1\}$$

From the properties, we have $S\{f^{(m)}(t)\} = \frac{1}{v^m}F(v) - \sum_{k=0}^{m-1} \frac{1}{v^{m-k+1}} f^{(k)}(0)$.
Using the properties of the Sawi transform: $\frac{1}{v}Y(v) - \frac{1}{v^2}Y(0) + \frac{Y(v)}{240} = \frac{1}{v}$
Solving for $Y(v)$: $\frac{1}{v}Y(v) + \frac{Y(v)}{240} = \frac{1}{v} + \frac{30}{v^2}$
$\left(1 + \frac{v}{240}\right)Y(v) = \frac{30}{v} + 1 \Rightarrow \left(1 + \frac{v}{240}\right)Y(v) = \frac{30+v}{v}$,
$Y(v) = \frac{30+v}{v\left(1+\frac{v}{240}\right)}$,
By partial fractions, $Y(v) = \frac{A}{v} + \frac{B}{v\left(1+\frac{v}{240}\right)}$ therefore, $A = 240\,, B = -210$ so
$Y(v) = \frac{240}{v} - \frac{210}{v\left(1+\frac{v}{240}\right)}$.
Inverse Sawi transform: To find $y(t)$, we take the inverse Sawi transform:
$y(t) = S^{-1}\left\{\frac{240}{v} - \frac{210}{v\left(1+\frac{v}{240}\right)}\right\} == S^{-1}\left\{\frac{240}{v} + \frac{210}{v\left(1+\frac{v}{240}\right)}\right\}$ We obtain $y(t) = 240 - 210e^{\frac{-t}{240}}$.
After 1 hour we can see $y(60) = 240 - 210e^{\frac{-60}{240}} = 240 - 210e^{-0.25} = 76.45\ kg$

Solution using properties of the Laplace transform (Joel L. Schiff, 1975)

Applying the Laplace transform to both sides of the equation yields:

$$\mathcal{L}\{y'(t)\} + \frac{1}{240}\mathcal{L}\{y(t)\} = \mathcal{L}\{1\}$$

Using the property $\mathcal{L}\{y'\} = sY(s) - y(0)$ and $\mathcal{L}\{1\} = \frac{1}{s}$, we obtain:

$$sY(s) - 30 + \frac{1}{240}Y(s) = \frac{1}{s}$$

Solving for $Y(s)$:

$$Y(s) = \frac{1}{s\left(s + \frac{1}{240}\right)} + \frac{30}{s + \frac{1}{240}}$$

Therefore, the Laplace transform of the solution is:

$$Y(s) = \frac{240}{s} - \frac{210}{s + \frac{1}{240}}$$

Applying the inverse Laplace transform gives the solution in the time domain:

$$y(t) = 240 - 210e^{-\frac{t}{240}}$$

In conclusion, the amount of salt in the tank asymptotically approaches 240 kg. At 60 minutes, approximately 76.45 kg of salt remains in the tank.
Figure (1) illustrates how the amount of salt in the tank increases exponentially from the initial value of 30 kg toward the steady-state value of 240 kg. At t = 60 minutes, the amount of salt is approximately 76.45 kg. As time progresses, the salt concentration approaches equilibrium, indicating that the inflow and outflow rates become balanced.

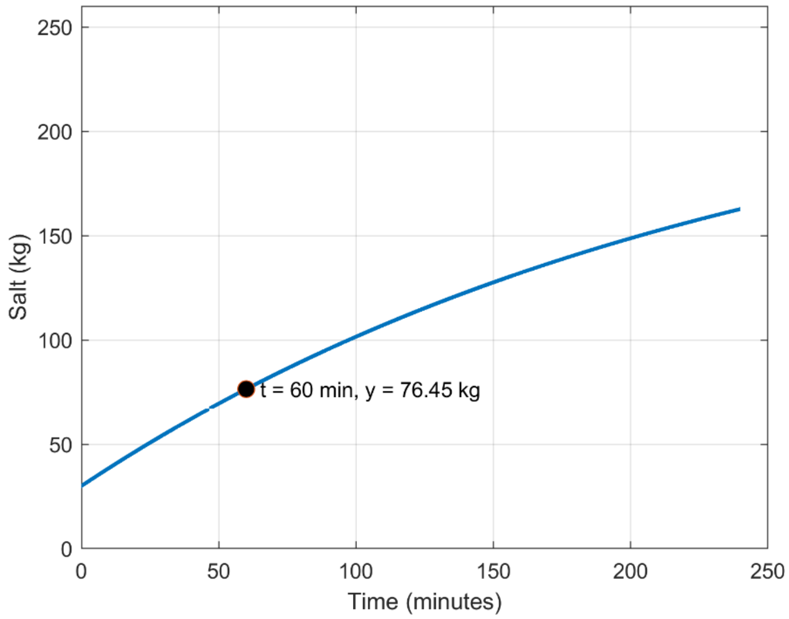

Figure: (1). Variation of salt amount in the tank with time

Problem 2

The municipal legislation states that any liquid that is released into the environment for the purpose of making "homemade" soap shall have a sodium chloride waste concentration of no more than 11.00 g/L. Liquid water that contains a lot of sodium chloride is the process's main by-product. The business only has one 15-liter tank available for holding waste. The waste tank held 750 grams of sodium chloride and 15 liters of water when filled. It is intended to pump fresh water into the tank at a rate of 2.0 liters per minute in order to maintain production and comply with local ordinances. Waste saltwater, which contains 25 grams of salt per liter, is pumped in at a rate of 1.5 liters per minute.

Waste is released at a rate of 3.5 liters per minute in order to maintain the solution level at 15 liters. Assume that in the flow diagram, A stands for the process' waste stream, B for fresh water, and C for the stream that is discharged into the environment. It is assumed that as the two streams, A and B, enter the tank, the concentration of chloride in the tank instantly changes to the exit concentration, $y(t)$. The tank system's material balance (sodium chloride) can be expressed as follows: Reduction by reaction plus input minus output equals accumulation.

Solution: Given that there is not a chemical reaction taking place in the storage tank, the above equation can be expressed as

$\frac{dy}{dt} = \left(25\frac{g}{L}\right)\left(1.5\frac{L}{min}\right) + \left(0\ \frac{g}{L}\right).\left(2\frac{L}{min}\right) - \left(y(t)\frac{g}{L}\right).\left(3.5\frac{L}{min}\right) + 0\ = 37.5\ \frac{g}{min} + 0 - 3.5x\ \frac{g}{min},$

Therefore, $\frac{dy}{dt} + 3.5y = 37.5$

For the initial condition of the ordinary differential equation at $t = 0$ the salt concentration in the tank was given as $50\frac{g}{L}$

we can use the Sawi transform to solve the first-order differential equation. The Sawi transform $S\{y(t)\}$ of a function $y(t)$ is defined as:

$S\{y(t)\} = \left(\frac{1}{v^2}\right)\int_0^\infty y(t)e^{-\frac{t}{v}}dt = Y(v), \qquad t > 0,$

Transform the differential equation: Applying the Sawi transform to both sides of the equation:

$S\left\{\frac{dy}{dt} + 3.5y\right\} = S\{37.5\}$

From the properties, we have $S\{f^{(m)}(t)\} = \frac{1}{v^m}F(v) - \sum_{k=0}^{m-1}\frac{1}{v^{m-k+1}}f^{(k)}(0).$

Using the properties of the Sawi transform: $\frac{1}{v}Y(v) - \frac{1}{v^2}Y(0) + 3.5Y(v) = \frac{37.5}{v}$
Solving for $Y(v)$: $\frac{1}{v}Y(v) + 3.5Y(v) = \frac{37.5}{v} + \frac{50}{v^2}$
$(1 + 3.5v)Y(v) = \frac{50}{v} + 37.5 \Longrightarrow (1 + 3.5v)Y(v) = \frac{50+37.5v}{v}$, $Y(v) = \frac{50+37.5v}{v(1+3.5v)}$,
By partial fractions, $Y(v) = \frac{A}{v} + \frac{B}{v(1+3.5v)}$ therefore, $A = \frac{75}{7}$,$B = \frac{275}{7}$ so
$Y(v) = \frac{\frac{75}{7}}{v} + \frac{\frac{275}{7}}{v(1+3.5v)}$.
Inverse Sawi transform: To find $y(t)$, we take the inverse Sawi transform:
$y(t) = S^{-1}\left\{\frac{75}{7v} + \frac{275}{7v(1+3.5v)}\right\} = S^{-1}\left\{\frac{75}{7v} + \frac{275}{7v(1+3.5v)}\right\}$ We obtain $y(t) = \frac{75}{7} + \frac{275}{7}e^{-3.5\text{t}}$.
Apply the Laplace transform to the ODE:

$$sY(s) - y(0) + 3.5Y(s) = \frac{37.5}{s}$$

Substitute $y(0) = 50$, and by rearranging the equation, we get:

$$Y(s) = \frac{37.5}{s(s + 3.5)} + \frac{50}{s + 3.5}$$

Decompose the first term into partial fractions:

$$Y(s) = \frac{75}{7s} + \frac{275}{7(s + 3.5)}$$

Apply the inverse Laplace transform:

$$y(t) = \frac{75}{7} + \frac{275}{7}e^{\{-3.5t\}}.$$

Figure 2 illustrates the temporal evolution of sodium chloride concentration in the 15-liter waste tank. Initially, the concentration is 50 g/L, and as fresh water is added while saltwater is discharged, the concentration decreases exponentially. As time progresses, the exponential term $e^{-3.5t}$ diminishes, causing the transient part of the solution to vanish, and the system approaches its steady-state condition. At approximately 1.5 minutes, the concentration nearly stabilizes at a constant value of $y_\infty = 10.7143g\backslash L$, which is represented by the red dashed line, while the blue curve shows the changing concentration over time.

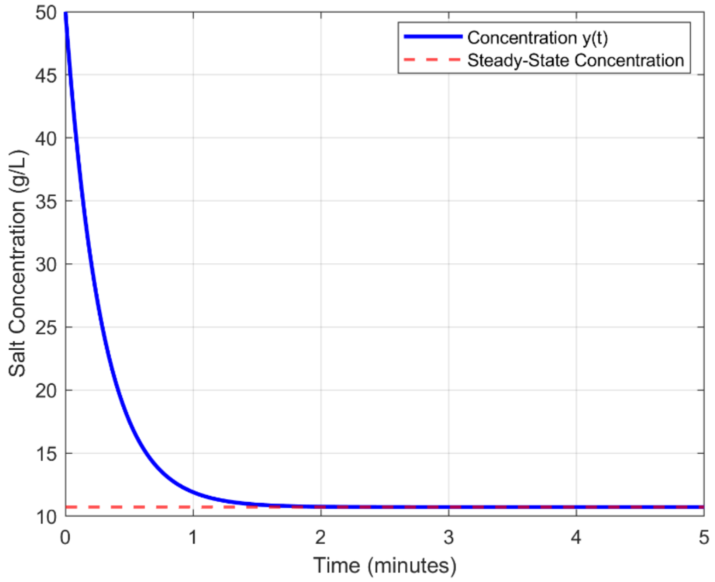

Figure: (2). Temporal evolution of sodium chloride concentration in waste tank

Problem 3

From the model of a first-order, single-step reversible reaction (Szabó et al., 2025), the dynamics of the concentration of species B can be described by the following equation (Szabó et al., 2025)

$$\frac{d^2B}{dt^2} + (k_1 + k_{-1})\frac{dB}{dt} = 0$$

Here, $\frac{dB}{dt}$ represents the instantaneous rate of change of B, while $\frac{d^2B}{dt^2}$ denotes the acceleration of this change. The constants k_1 and k_2 correspond to the forward and reverse reaction rate constants, respectively. This formulation captures the temporal evolution of B in a single-step reversible reaction and provides a foundation for further kinetic analysis.

Assuming that b_0 is the concentration of the substance at the initial time ($B(0) = b_0$) and that the initial rate of change of the concentration is b_1, which represents the initial speed of formation (or consumption) of the substance at the start of the reaction ($B'(0) = b_1$).

We apply the Sawi transform to the given differential equation.

$$S\left\{\frac{d^2B}{dt^2}\right\} + (k_1 + k_{-1})S\left\{\frac{dB}{dt}\right\} = 0$$

$$\Rightarrow \frac{1}{v^2}\hat{B}(v) - \frac{1}{v^3}B(0) - \frac{1}{v^2}B'(0) + (k_1 + k_{-1})\left(\frac{1}{v}\hat{B}(v) - \frac{1}{v^2}B(0)\right) = 0$$

By substituting the initial conditions and rearranging the previous equation, we obtain:

$\left(\frac{1}{v^2} + \frac{k_1+k_{-1}}{v}\right)\hat{B}(v) - \left(\frac{1}{v^3} + \frac{k_1+k_{-1}}{v^2}\right)b_0 - \frac{1}{v^2}b_1 = 0$, That is

$$\hat{B}(v) = \frac{1}{v}b_0 + \frac{1}{1+v(k_1+k_{-1})}b_1$$

$$\Rightarrow \quad \hat{B}(v) = \frac{1}{v}b_0 + \left(\frac{1}{(k_1+k_{-1})v} - \frac{1}{(k_1+k_{-1})v} + \frac{1}{1+v(k_1+k_{-1})}\right)b_1$$

That is $\hat{B}(v) = \frac{1}{v}b_0 + \left(\frac{1}{(k_1+k_{-1})v} - \frac{1}{1+v(k_1+k_{-1})}\right)b_1$

By applying the inverse Sawi transform, the following result is obtained

$B(t) = b_0 + \frac{b_1}{k_1+k_{-1}}\left(1 - e^{-(k_1+k_{-1})t}\right)$.

Solution by the Laplace transform.

Taking the Laplace transform $\mathcal{L}\{B(t)\} = \breve{B}(s)$. Using standard derivative rules

$(s^2 + (k_1 + k_{-1})s)\breve{B}(s) - s\,b_0 - b_1 - (k_1 + k_{-1})\,b_0 = 0$. That is $\breve{B}(s) = \frac{b_0}{s} + \frac{b_1}{s^2+(k_1+k_{-1})s}$.

Partial fractions (or known pairs) give $\frac{b_1}{s^2+(k_1+k_{-1})s} = \frac{b_1}{k_1+k_{-1}}\left(\frac{1}{s} - \frac{1}{s+(k_1+k_{-1})}\right)$. By applying the inverse Laplace transform, we obtain: $B(t) = b_0 + \frac{b_1}{k_1+k_{-1}}\left(1 - e^{-(k_1+k_{-1})t}\right)$

From the solution, we deduce that as time ($t \to \infty$), the exponential term $e^{-(k_1+k_{-1})t}$, which represents the transient phase before reaching the steady state, vanishes, and the concentration of the substance $B(t)$ gradually approaches the constant value at the steady state:

$B_\infty = b_0 + \frac{b_1}{k_1+k_{-1}}$.

In this example, it is assumed that the initial concentration of species B at time zero is $B(0) = 1\,mol/L$, and the initial rate of change of the concentration at the start is $B'(0) = 0.5\,mol/L$. Using these values and the reaction rate constants $k_1 = 0.3s^{-1}$ and $k_{-1} = 0.1s^{-1}$, the plot illustrates the temporal evolution of B in a first-order single-step reversible reaction. The curve shows that the concentration starts from the initial value and gradually increases toward equilibrium, with the red marker in the figure representing the initial point $B(0) = 1\,mol/L$ at time zero.

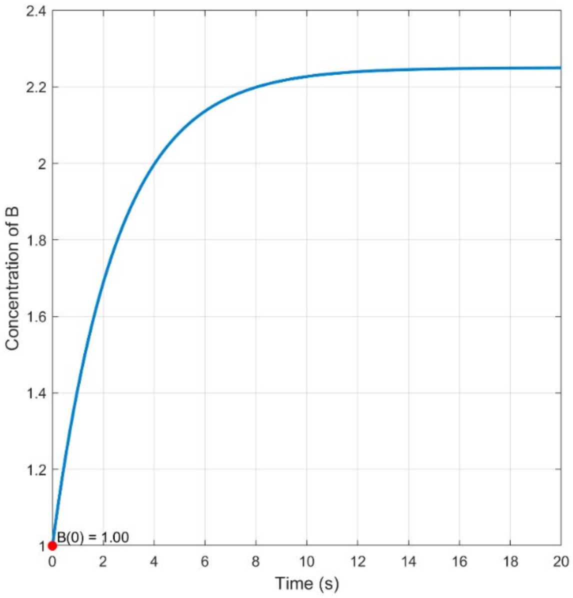

Figure: (3). Temporal evolution of species B in a first-order single-step reversible reaction

Results and Discussion

The results of the study show that the Sawi transform is a dependable and effective technique for obtaining analytical solutions to kinetic models of chemical reactions. Compared to the well-known Laplace transform, the Sawi transform is more efficient and easier to apply when solving first- and second-order differential equations. The same exact solutions produced by both methods validated the accuracy of the Sawi transform in converting complex mathematical structures into solvable algebraic forms within the time domain. However, this study found that one of the main drawbacks is the Sawi transform's current application to linear models. For extremely nonlinear functions, which frequently arise in more complex chemical dynamics, the transform may need to be further adjusted or utilized in conjunction with numerical approaches. Despite this, because of its dependability in solving common chemical reaction models, it is a useful alternative for researchers in the field of mathematical chemistry.

CONCLUSION

This study demonstrates the effectiveness of the Sawi transform in solving mathematical models in chemical sciencess. A comparison with the Laplace transform shows that both methods provide the same level of accuracy in the final results, while maintaining simplicity and avoiding complex computations. This highlights the capability of the Sawi transform to deliver precise and efficient solutions for chemical mixing problems. Furthermore, the results suggest that the Sawi transform could be extended in the future to address a wide range of challenging problems in science, technology, and medicine through mathematical modeling.

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ETHICS

The authors declare that no ethical issues are associated with this manuscript.

Duality of interest

The authors declare that they have no duality of interest associated with this manuscript.

Author contributions

Contributions were equal between the authors.

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