Al-Mukhtar Journal of Basic Sciences 23 (3): 139-154, 2025
Doi: https://doi.org/10.54172/pw8nzz89

Research Article **Open Access**

Inclined load Effects oN Thermoelastic Porous Medium with Temperature Dependent and Rotating Medium Under Different Theories

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Received:
14 August 2025

Accepted:
28 October 2025

Publish online:
31 December 2025

Abstract

The aim of the present paper is to study the effect of inclined load, temperature dependent, and rotation thermo-elastic solid with voids. The problem is formulated in the context of the different theories of generalized thermoelasticity: Lord-Schulman with one relaxation time, Green-Lindsay with two relaxation times, and the coupled theory. The exact expression of the physical quantity for the temperature, stress, and displacement components are found is obtained by normal mode analysis and illustrated graphically by comparison and discussion in different values of the angle of inclination also the presence and the absence of rotation and temperature dependent.

Keywords: Inclined load, Rotation, Voids, Normal Mode Analysis, temperature dependent, Generalized Thermoelasticity.

INTRODUCTION

The subject of generalized thermoelasticity has drawn the attention of researchers due to its relevance in many practical applications. In contrast to the coupled thermoelasticity theory based on a parabolic heat equation (Biot, 1956), which predicts an infinite speed of the propagation of heat. The first generalization, for isotropic bodies, is because of (Lord & Shulman, 1967), introduced the generalized thermoelastic theory withone relaxation period, which replaces the standard Fourier law by posing a new law of heat conduction. The second generalization is known as the theory of temperature rate dependent thermoelasticity, and was proposed by (Green & Lindsay, 1972).

A nonlinear theory of elastic material with voids was developed by (Cowin & Nunziato, 1983; Nunziato & Cowin, 1979) developed a theory of linear elastic materials with voids. (Ieşan, 1986) developed a proposition of thermoelastic accoutrements with voids. lately, (Aoudai, 2010) deduced a proposition of thermoelastic prolixity Material with voids. (Puri & Cowin, 1985) studied the geste of aeroplane swells in a direct elastic material with voids. (Dhaliwal & Wang, 1995) developed a heat flux dependent proposition of hermoelasticity with voids.

The effect of an inclined load on a functionally graded, temperature-dependent thermoelastic material was analyzed by (Barak & Dhankhar, 2022).

(Abouelregal & Alesemi, 2022; Deswal et al., 2019; Othman et al., 2018) have studied many problems in inclined load thermoelastic materials. (Said, 2024) studied the lmpact of rotation and inclined load a nonlocal fiber-renforced thermoelastic half-space via simple-phase-lag model , (Said et al., 2024) studied Effect of an inclined load on a nonlocal fiber-reinforced visco-thermoelastic solid via a dual-phase-lag model.(Kumar & Ailawalia, 2005; Kumar & Gupta, 2010) studied variety of issues arising from inclined loads in the micropolar elastic media. (Abouelregal & Zenkour, 2016) proposed a two-temperature theory, To examine the thermally insulated stress-free surface of a thermoelastic solid half-space as a result of an inclined load. (Lata & Kaur, 2019) studied Efect of rotation and inclined load on transversely isotropic magneto thermoelastic solid. (Deswal et al., 2020) discussed disturbances in an initially stressed fber-reinforced orthotropic thermoelastic medium due to inclined load. (Alharbi, 2021) introduced two temperature theory on a micropolar thermoelastic media under the efect of inclined load with voids.(Othman et al., 2023) studied Efect of initial stress and inclined load on generalized micropolar thermoelastic with three-phase-lag model. (Sheokand et al., 2024) discussed Infuence of variable thermal conductivity and inclined load on a nonlocal.

The thermal stress in a material with temperature-dependent properties was studied extensively by (Noda, 1986) . (Abbas, 2014) have used the eigenvalue approach in a three-dimensional generalized thermoelastic interaction with temperature-dependent material properties. (Othman et al., 2013) have investigated the generalized thermoelastic medium with temperature dependent properties for different theories under the effect of gravity field. (Othman et al., 2014) studied the effect of rotation on the problem of fiber-reinforced under generalized magneto thermoelasticity subject to thermal loading due to laser pulse: a comparison of different theories.

Othman and (Othman & Edeeb, 2018) studied the effect of rotation and temperature dependent on thermoelastic medium with voids under three theories. (Othman, 2011) discussed the state-space approach to the generalized thermoelastic problem with temperature-dependent elastic medium and internal heat sources.

BASIC EQUATIONS OF THE PROBLEM

We consider a homogeneousv oids thermoelastic half- space under the influence of rotation and temperature dependent . All the considered quan- tities are functions of the time variable t and of the coordinates x and z .

Since the medium is rotating uniformly with an angular velocity $\boldsymbol{\Omega} = \Omega \mathbf{n}$ where $\mathbf{n}$ is a unit vector representing the direction of the axis of the rotation, the equation of motion in the rotating frame of reference has two additional terms (Schoenberg & Censor, 1973): centripetal acceleration $\boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{u})$ due to time varying motion only and Corioli's acceleration $2\boldsymbol{\Omega} \wedge \dot{\boldsymbol{u}}$, then the fundamental equations of the generalized thermoelasticity are

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad i, j = 1,3. \tag{1}$$

$$\sigma_{ij} = [\lambda e_{kk} + b\phi - \beta\,(1 + \upsilon_0 \frac{\partial}{\partial t})T\,]\delta_{ij} + \mu\, e_{ij}\,, \tag{2}$$

then the equation of motion in a rotating frame of reference is

$$\sigma_{ij,j} = \rho[\ddot{u}_i + \{\boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \boldsymbol{u}\}_i + 2(\boldsymbol{\Omega} \times \dot{\boldsymbol{u}})_i\,], \quad i, j = 1,3. \tag{3}$$

Where σ_{ij} are the components of stress tensor, e_{ij} are the components of strain, $\lambda,\ \mu$ are the lame's constants, $\beta = (3\lambda + 2\mu)\alpha_t$ such that α_t is the coefficient of thermal expansion, δ_{ij} is the Kronecker delta, and $i, j = x, z$.

The displacement components have the following form $\boldsymbol{u} = (u_1, 0, u_3)$.

The dynamical equations of an elastic medium are given by

$$\mu\,\nabla^2 u_1 + (\lambda+\mu)\frac{\partial e}{\partial x} + b\,\frac{\partial \phi}{\partial x} - \beta(1+\upsilon_0\frac{\partial}{\partial t})\nabla\frac{\partial T}{\partial x} = \rho[\ddot{u}_1 - \Omega^2 u_1 - 2\Omega\dot{u}_3], \tag{4}$$

$$\mu\,\nabla^2 u_3 + (\lambda+\mu)\frac{\partial e}{\partial z} + b\,\frac{\partial \phi}{\partial z} - \beta(1+\upsilon_0\frac{\partial}{\partial t})\nabla\frac{\partial T}{\partial z} = \rho[\ddot{u}_3 - \Omega^2 u_3 + 2\Omega\dot{u}_1], \tag{5}$$

The equation of voids is

$$\alpha\,\nabla^2\phi - b\,e - \xi\phi - \omega_0 b\frac{\partial \phi}{\partial t} + m(1+\upsilon_0\frac{\partial}{\partial t})T = \rho\,\chi\frac{\partial^2\phi}{\partial t^2}, \tag{6}$$

The heat conduction equation,

$$K\nabla^2 T = \rho C_E(1+\tau_0\frac{\partial}{\partial t})\dot{T} + \beta T_0(1+n_0\tau_0\frac{\partial}{\partial t})\dot{e} + mT_0(1+n_0\tau_0\frac{\partial}{\partial t})\dot{\phi}. \tag{7}$$

The components of stress tensor are

$$\sigma_{xx} = \lambda(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}) + \mu\frac{\partial u_1}{\partial x} + b\phi - \beta(1+\upsilon_0\frac{\partial}{\partial t})T, \tag{8}$$

$$\sigma_{zz} = \lambda(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}) + \mu\frac{\partial u_3}{\partial z} + b\phi - \beta(1+\upsilon_0\frac{\partial}{\partial t})T, \tag{9}$$

$$\sigma_{xz} = \lambda(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}). \tag{10}$$

Eqs. (4) and (7) are the field equations of the generalized linear thermoelasticity for a rotating media, applicable to the coupled theory, three generalizations, as follows:

1. The coupled (CD) theory, when
   $n_0 = \tau_0 = \upsilon_0 = 0.$
2. Lord-Shulman (L-S) theory, when
   $n_0 = 1, \quad \upsilon_0 = 0, \;\; \tau_0 > 0.$
3. Green-Lindsay (G-L) theory, when
   $n_0 = 0, \quad \upsilon_0 \geq \tau_0 > 0.$

Where K is the thermal conductivity, ρ is the density, C_E is the specific heat at constant strain, $\tau_0,\ \upsilon$ are the thermal relaxation times, n_1, n_0 are parameters, ϕ is the change in the volume fraction field, T_0 is the reference temperature is chosen so that $|(T-T_0)/T_0| \ll 1,$, $i, j = x, z$. $\alpha, b, \xi, \omega_0, m, \chi$ are the material constants due to the presence of voids.

Assume that

$\lambda = \lambda_0 f(T), \quad \mu = \mu_0 f(T), \quad \xi = \xi_0 f(T), \quad m = m_0 f(T), \quad b = b_0 f(T), \quad k = k_0 f(T),$
$\beta = \beta_0 f(T).$

Where $f(T)$ is a given dimensionless function of temperature such that $f(T) = 1 - \alpha^* T_0$ (α^* is an empirical material constant). In the case of temperature independent modulus of elasticity, $f(T) = 1.$

In order to facilitate the solution, the following dimensionless quantities are introduced :

$$(x', z') = \frac{\omega_1^*}{c_0}(x, z), \quad (u_1', u_3') = \frac{\omega_1^*}{c_1}(u_1, u_3), \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\mu_0}\sigma_{ij}, \quad \phi' = \frac{\omega_1^*\chi_0}{c_1^2}\phi, \quad T' = \frac{T}{T_0},$$

$$t' = \omega_1^* t, \ \Omega' = \frac{\Omega}{\omega_1^*}, \ c_1^2 = \frac{\lambda_0 + 2\mu_0}{\rho}, \ \omega_1^* = \frac{\rho C_E c_1^2}{k}, \ \upsilon_0' = \omega_1^*\upsilon_0, \ \tau_0' = \omega_1^*\tau_0, \tag{12}$$

From Eq. (12) in to Eqs. (4)-(7) we get

$$\nabla^2 u_1 + h_1 \frac{\partial e}{\partial x} + h_2 \frac{\partial \phi}{\partial x} - h_3 (1+\upsilon_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial x} = h_4 [\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 - 2\Omega \frac{\partial u_3}{\partial t}], \qquad (13)$$

$$\nabla^2 u_3 + h_1 \frac{\partial e}{\partial z} + h_2 \frac{\partial \phi}{\partial z} - h_3 (1+\upsilon_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial z} = h_4 [\frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 + 2\Omega \frac{\partial u_1}{\partial t}], \qquad (14)$$

$$\nabla^2 \phi - \eta_5 e - h_6 \phi - h_7 \frac{\partial \phi}{\partial t} + h_8 (1+\upsilon_0 \frac{\partial}{\partial t}) T = h_9 \frac{\partial^2 \phi}{\partial t^2}, \qquad (15)$$

$$\varepsilon_1 \nabla^2 T - h_{10} (1 + n_0 \tau_0 \frac{\partial}{\partial t}) \frac{\partial \phi}{\partial t} = (1+\tau_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial t} + \varepsilon_2 (1 + n_0 \tau_0 \frac{\partial}{\partial t}) \frac{\partial e}{\partial t}. \qquad (16)$$

Also, the constitutive Eqs. (8)-(10) reduces to

$$\sigma_{xx} = h_{11} [\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}] + \frac{2}{A^*} \frac{\partial u_1}{\partial x} + h_{12} \phi - h_{13} (1+\upsilon_0 \frac{\partial}{\partial t}) T, \qquad (17)$$

$$\sigma_{zz} = h_{11} [\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}] + \frac{2}{A^*} \frac{\partial u_3}{\partial z} + h_{12} \phi - h_{13} (1+\upsilon_0 \frac{\partial}{\partial t}) T, \qquad (18)$$

$$\sigma_{xz} = \frac{1}{A^*} [\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}]. \qquad (19)$$

Where

$$h_1 = \frac{\lambda_0 + \mu_0}{\mu_0}, \quad h_2 = \frac{b_0 c_1^2}{\mu_0 \omega_1^{*2} \chi_0}, \quad h_3 = \frac{\beta_0 T_0}{\mu_0}, \quad h_4 = \frac{A^* \rho c_1^2}{\mu_0}, \quad h_5 = \frac{b_0 \chi_0}{\alpha_0}, \quad h_6 = \frac{\xi_0 c_1^2}{\alpha_0 \omega_1^{*2}},$$

$$h_7 = \frac{\omega_{10} c_1^2}{\alpha_0 \omega_1^*}, \quad h_8 = \frac{m_0 T_0 \chi_0}{\alpha_0}, \quad h_9 = \frac{\rho c_1^2 \chi_0}{\alpha_0}, \quad h_{10} = \frac{m_0 c_1^2}{A^* \rho C_E \omega_1^{*2} \chi_0}, \quad \varepsilon_1 = \frac{K_0 \omega_1^*}{A^* \rho c_1^2 C_E},$$

$$\varepsilon_2 = \frac{\beta_0}{A^* C_E e}, \quad h_{11} = \frac{\lambda_0}{A^* \mu_0}, \quad h_{12} = \frac{b_0 c_1^2}{A^* \mu_0 \omega_1^{*2} \chi_0}, \quad h_{13} = \frac{\beta_0 T_0}{A^* \mu_0}, \quad A^* = \frac{1}{f(T)} = \frac{1}{(1-\alpha^* T_0)}.$$

Introducing the displacement potentials $R(x,z,t)$ and $\psi(x,z,t)$ which related to the displacement components by the relations:

$$u_1 = R_{,x} + \psi_{,z}, \qquad u_3 = R_{,z} - \psi_{,x}. \qquad (20)$$

$$e = \nabla^2 R, \quad \text{and} \quad (\frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x}) = \nabla^2 \psi. \qquad (21)$$

By substituting from Eq. (21) in Eqs. (13)-(16), this yields

$$[(1+h_1)\nabla^2 - h_4 (\frac{\partial^2}{\partial t^2} - \Omega^2)] R - 2h_4 \Omega \psi + h_2 \phi - h_3 (1+\upsilon_0 \frac{\partial}{\partial t}) T = 0, \qquad (22)$$

$$2h_4 \Omega \frac{\partial R}{\partial t} + [\nabla^2 - h_4 (\frac{\partial^2}{\partial t^2} - \Omega^2)] \psi = 0, \qquad (23)$$

$$-h_5 \nabla^2 R + (\nabla^2 - h_6 - h_7 \frac{\partial}{\partial t} - h_9 \frac{\partial^2}{\partial t^2}) \phi + h_8 (1+\upsilon_0 \frac{\partial}{\partial t}) T = 0, \qquad (24)$$

$$\varepsilon_1 \nabla^2 T - h_{10} (1 + n_0 \tau_0 \frac{\partial}{\partial t}) \frac{\partial \phi}{\partial t} = (1+\tau_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial t} + \varepsilon_2 (1 + n_0 \tau_0 \frac{\partial}{\partial t}) \frac{\partial e}{\partial t}. \qquad (25)$$

$$-\varepsilon_2 (\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}) \nabla^2 R - h_{10} (1 + n_0 \tau_0 \frac{\partial}{\partial t}) \frac{\partial \phi}{\partial t} + \varepsilon_1 \nabla^2 T - (1+\tau_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial t} = 0. \qquad (26)$$

The Method of Solution of The Problem

The used analytical method to obtain the exact expressions for the physical variables is the normal mode analysis. The solution of considered physical variables can be decomposed in terms of normal mode as following form

$$[R, \psi, T, \phi, \sigma_{ij}, F_1, F_2, f](x,z,t) = [R^*, \psi^*, T^*, \phi^*, \sigma_{ij}^*, F_1^*, F_2^*](z) \exp[i(\omega t + a x)], \qquad (27)$$

Where ω is the complex time constant, $i=\sqrt{-1}$ and a is the wave number in x-direction and $R^*, \psi^*, T^*, \phi^*, \sigma_{ij}^*, F_1^*$, and F_2^* are the amplitudes of the field quantities..

Using Eq. (27) in Eqs. (22)-(26), lead to

$$(\mathrm{D}^2 - S_2)R^* - S_3\psi^* + S_4\phi^* - S_5T^* = 0, \tag{28}$$

$$\mathrm{S}_6\mathrm{R}^* + (\mathrm{D}^2 - S_7)\,\psi^* = 0, \tag{29}$$

$$(-A_5\mathrm{D}^2 + S_8)R^* + (\mathrm{D}^2 - S_9)\phi^* + S_{10}T^* = 0, \tag{30}$$

$$S_{11}(\mathrm{D}^2 - a^2)R^* - S_{12}\phi^* + (\mathrm{D}^2 - S_{13})T^* = 0. \tag{31}$$

Using Eqs. (20) and (27) in Eqs. (17)-(19), we get

$$\sigma_{xx}^* = h_{11}(\mathrm{D}^2 - a^2)R^* + \frac{2}{A^*}iau_1^* + h_{12}\phi^* - h_{13}(1 + i\upsilon_0\omega)T^*, \tag{32}$$

$$\sigma_{zz}^* = h_{11}(\mathrm{D}^2 - a^2)R^* + \frac{2}{A^*}\mathrm{D}u_3^* + h_{12}\phi^* - h_{13}(1 + i\upsilon_0\omega)T^*, \tag{33}$$

$$\sigma_{xz}^* = \frac{1}{A^*}[\mathrm{D}u_1^* + iau_3^*]. \tag{34}$$

Where,

$$S_1 = 1 + h_1,\quad S_2 = \frac{S_1a^2 - h_4(\omega^2 + \Omega^2)}{S_1},\quad S_3 = \frac{2h_4\Omega}{S_1},\quad S_4 = \frac{h_2}{S_1},\quad S_5 = \frac{h_3(1 + i\upsilon_0\omega)}{S_1},$$

$$S_6 = 2h_4 i\,\omega\Omega,\quad S_7 = a^2 - h_4(\omega^2 + \Omega^2),\quad S_8 = h_5a^2,\quad S_9 = a^2 + h_6 + \mathrm{i}h_7\omega - h_9\omega^2,$$

$$S_{10} = h_8(1 + i\upsilon_0\omega),\quad S_{11} = \frac{-\varepsilon_2(\mathrm{i}\omega - n_0\tau_0\omega^2)}{\varepsilon_1},\quad S_{12} = \frac{h_{11}(\mathrm{i}\omega - n_0\tau_0\omega^2)}{\varepsilon_1},$$

$$S_{13} = \frac{\varepsilon_1a^2 + \mathrm{i}\omega - \tau_0\omega^2}{\varepsilon_1},\ \mathrm{D} = \frac{\mathrm{d}}{\mathrm{dz}}.$$

Eliminating $\psi^*(x)$, $\phi^*(x)$ and $T^*(x)$ between Eqs. (28)-(31), this lead to the following eighth order ordinary differential equation satisfied with $R^*(x)$:

$$[\mathrm{D}^8 - B_1\mathrm{D}^6 + B_2\mathrm{D}^4 - B_3\mathrm{D}^2 + B_4]R^*(\mathrm{z}) = 0. \tag{35}$$

Where

$$B_1 = S_{13} + S_9 + S_7 + S_2 - A_5S_4 - S_5S_{11},$$

$$\begin{aligned}B_2 = {} & S_9S_{13} + S_{10}S_{12} + S_7S_{13} + S_7S_9 + S_2S_{13} + S_2S_9 + S_2S_7 + S_6S_3 - A_5S_4S_{13} \\ & - S_4S_8 + S_4S_{10}S_{11} - A_5S_4S_7 - A_5S_5S_{12} - S_5S_{11}a^2 - S_5S_9S_{11} - S_5S_7S_{11},\end{aligned}$$

$$\begin{aligned}B_3 = {} & S_7S_9S_{13} + S_7S_{10}S_{12} + S_2S_9S_{13} + S_2S_{10}S_{12} + S_2S_7S_{13} + S_2S_7S_9 + S_3S_6S_{13} \\ & + S_3S_6S_9 - S_4S_8S_{13} + S_4S_{10}S_{11}a^2 - A_5S_4S_7S_{13} - S_4S_7S_8 + S_4S_7S_{10}S_{11} \\ & - S_5S_8S_{12}^2 - S_5S_9S_{11}a^2 - A_5S_5S_7S_{12} - S_5S_7S_{11}a^2 - S_5S_7S_9S_{11},\end{aligned}$$

$$\begin{aligned}B_4 = {} & S_2S_7S_9S_{13} + S_2S_7S_{10}S_{12} + S_3S_6S_9S_{13} + S_3S_6S_{10}S_{12} - S_4S_7S_8S_{13} + S_4S_7S_{10}S_{11}a^2 \\ & - S_5S_7S_8S_{12} - S_5S_7S_9S_{11}a^2.\end{aligned}$$

Equation (34) can be factored as

$$[(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)(\mathrm{D}^2 - k_3^2)(\mathrm{D}^2 - k_4^2)]R^*(\mathrm{z}) = 0. \tag{36}$$

The solution of Eq. (35) which is bounded as $z \to \infty$, is given by:

$$R^* = \sum_{n=1}^{4} M_n e^{-k_n z} , \tag{37}$$

$$\psi^* = \sum_{n=1}^{4} H_{1n} M_n e^{-k_n z} , \tag{38}$$

$$\phi^* = \sum_{n=1}^{4} H_{2n} M_n e^{-k_n z} , \tag{39}$$

$$T^* = \sum_{n=1}^{4} H_{3n} M_n e^{-k_n z} . \tag{40}$$

The solutions of the characteristic equation of equation (35) are $k_n^2 (n = 1, 2, 3, 4)$ and $M_n \; (n = 1, 2, 3, 4)$, are some parameters.

Substituting from equations (37) and (38) in (20) to get

$$u_1^* = \sum_{n=1}^{4} H_{4n} M_n e^{-k_n z} , \tag{41}$$

$$u_3^* = \sum_{n=1}^{4} H_{5n} M_n e^{-k_n z} , \tag{42}$$

From Eqs. (37)-(42) in (32)-(34) to obtain the following expressions for the stress tensor components

$$\sigma_{xx}^* = \sum_{n=1}^{4} H_{6n} M_n e^{-k_n z} , \tag{43}$$

$$\sigma_{zz}^* = \sum_{n=1}^{4} H_{7n} M_n e^{-k_n z} , \tag{44}$$

$$\sigma_{xz}^* = \sum_{n=1}^{4} H_{8n} M_n e^{-k_n z} . \tag{45}$$

Where,

$$H_{1n} = \frac{-S_6}{(k_n^2 - S_7)}, \qquad H_{2n} = \frac{-[S_5(-h_5 k_n^2 + S_8) + S_{10}(k_n^2 - S_2 - S_3 H_{1n}]}{[S_5(k_n^2 - S_9) + S_4 S_{10}]},$$

$$H_{3n} = \frac{k_n^2 - S_2 - S_3 H_{1n} + S_4 H_{2n}}{S_5}, \qquad H_{4n} = ia - k_n H_{1n,}, \qquad H_{5n} = -(k_n + iaH_{1n}),$$

$$H_{6n} = h_{11}(iaH_{4n} - k_n H_{5n}) + 2iaH_{4n} + h_{12} H_{2n} - h_{13}(1 + i\upsilon_0 \omega) H_{3n},$$

$$H_{7n} = h_{11}(iaH_{4n} - k_n h_{5n}) - 2k_n h_{5n} + h_{12} H_{2n} - h_{13}(1 + i\upsilon_0 \omega) H_{3n}, \quad H_{8n} = (-k_n H_{4n} + i\,a H_{5n}).$$

BOUNDARY CONDITIONS

We consider an inclined load p acting in the direction who make an angle θ with the direction of the x-axis:

$$\sigma_{zz} = F_1 e^{i(\omega t + ax)} = -(\mathrm{P}\cos\theta) e^{i(\omega t + ax)}, \qquad \sigma_{xz} = F_2 e^{i(\omega t + ax)} = -(\mathrm{P}\sin\theta) e^{i(\omega t + ax)},$$

$$\frac{\partial \phi}{\partial z} = 0, \quad T = f(x, t). \tag{46}$$

By Substituting the expression of the variables considered into the above boundary conditions (46), we can obtain the following equations satisfied by the parameters

$$\sum_{n=1}^{4} H_{7n} M_n = F_1 = -(\text{P}\cos\theta) e^{i(\omega t + a x)}, \quad (47)$$

$$\sum_{n=1}^{4} H_{8n} M_n = F_2 = -(\text{P}\sin\theta) e^{i(\omega t + a x)}, \quad (48)$$

$$\sum_{n=1}^{4} -k_n H_{2n} M_n = 0, \quad (49)$$

$$\sum_{n=1}^{4} H_{3n} M_n = f^{*} e^{i(\omega t + a x)}. \quad (50)$$

Solving Eqs. (47 - 50) for $M_n\,(n = 1, 2, 3, 4)$ by using the inverse of matrix method as follows:

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} H_{71} & H_{72} & H_{73} & H_{74} \\ H_{81} & H_{82} & H_{83} & H_{84} \\ -k_1 H_{21} & -k_1 H_{22} & -k_1 H_{23} & -k_1 H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \end{pmatrix}^{-1} \begin{pmatrix} F_1^{*} \\ F_2^{*} \\ 0 \\ f^{*} \end{pmatrix}. \quad (51)$$

Numerical Results

Copper material was chosen for purposes of numerical evaluations and the constants of the problem were taken as follows:

$\lambda = 7.76\times10^{10}\ N.m^{-2}$, $\mu = 3.86\times10^{10}\ kg.m^{-1}.s^{-2}$, $K = 386\,w.m^{-1}.k^{-1}$, $\alpha_t = 1.78\times10^{-5}\,k^{-1}$,

$\rho = 8954\ kg.m^{-3}$, $C_E = 383.1\,J.kg^{-1}.k^{-1}$, $T_0 = 293\,K$.

$\beta = 2.68\times10^{6} N/m^2$ deg, $\omega_1^{*} = 3.58\times10^{11}/s$.

The voids parameters are

$\chi = 1.753\times10^{-15} m^2$, $\xi = 1.475\times10^{10} N/m^2$, $b = 1.13849\times10^{10} N/m^2$,

$\alpha = 3.688\times10^{-5} N$, $m = 2\times10^{6} N/m^2$ deg, $\omega_0 = 0.0787\times10^{-3} N/m^2 s$.

The comparisons were carried out for

$x = 0.5$, $\theta = 30$, $\omega = \zeta_0 + i\zeta_1$, $\zeta_0 = 1$, $\xi_1 = -1.1$ $\text{P} = 0.9$ $f = 1.0$, $\tau_0 = 1s$, $\upsilon_0 = 1.5$

$a = 0.1$, $\Omega = 1$, $0 \le z \le 4$.

The computations are carried out for a value of time $t = 0.01$s. The variations of the thermodynamic temperature T, the fraction field ϕ, the displacement components with distance in three theories, for these cases

(i) Figure s 1-5 in the absence $\Omega = 0$ and presence $\Omega = 1$ of the rotation, in the presence of a temperature dependent and $\theta = 30.$.

(ii) Figure s 6-10 for the absence $\alpha^{*} = 0$ with and presence $\alpha^{*} = 0.00051$ while $\Omega = 1$.

(iii) Figure s 11-15 for different values of the Inclined load dependent properties in the case of two different values of $\theta = 30, 15.$ while $\Omega = 1$.

Figure s 9 and 10 display the 3D surface curves for the stress components within the framework of the (G-L) theory. Different Figure s can be used to examine how different variables affect the vertical component of displacement. During wave propagation, all physical quantities are in motion; hence the vertical displacement has a significant impact on the curves that are produced.

Figure 1 illustrates thatthe distribution of the horizontal displacement u_1 begins from a positive value in the case of $(\Omega = 0,1)$. In the context of the three theories, we notice that the distribution of u_1 is increased with the decreasing of the rotation for $z > 0$, the distribution of u_1 is inversely proportional to the rotation.

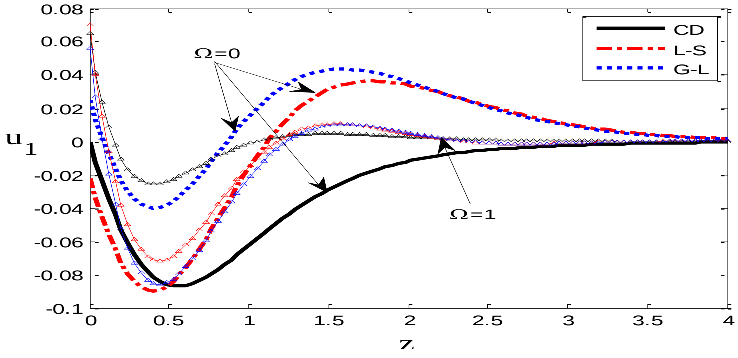

Figure (1). Variation displacement distribution u_1 in the absence and presence of rotation

Figure 2 demonstrates that the distribution of the temperature T always begins from zero and satisfies the boundary conditions (which is the same point) in the case of $(\Omega = 0,1)$ in the context of the three theories, and we notice that the rotation has a small effect in the distribution of temperature T.

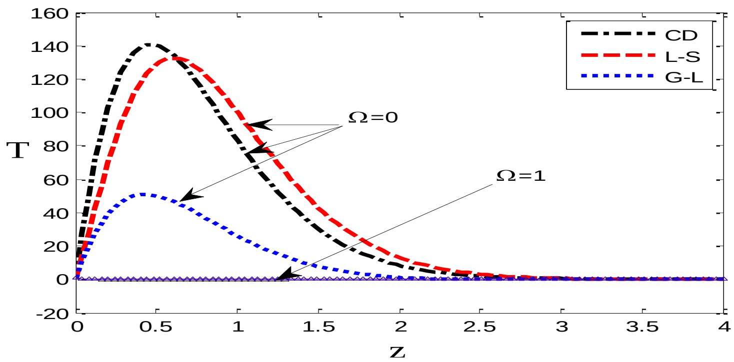

Figure (2). Variation of the thermal temperature T in the absence and presence of rotation

Figure 3 explains the distribution of the change in the volume fraction field ϕ in the context of three theories in the case of $\Omega = 0$, and $\Omega = 1$, it observed that rotation has a great effect on the distribution of ϕ, while the distribution of ϕ is decreasing with the increase of the rotation value for $z > 0$.

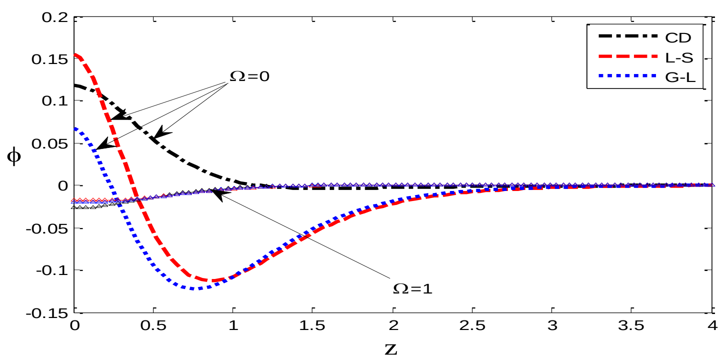

Figure (3) . Variation of volume fraction field ϕ in the absence and presence of rotation

Figure 4 depict the distribution of the stress tensor components σ_{zz} in the context of the three theories for $\Omega = 0$, and t $\Omega = 1$, , it observed that the distributions of σ_{zz} is inversely proportional to the rotation while they decreasing with the increase of the rotation value for $z > 0$.

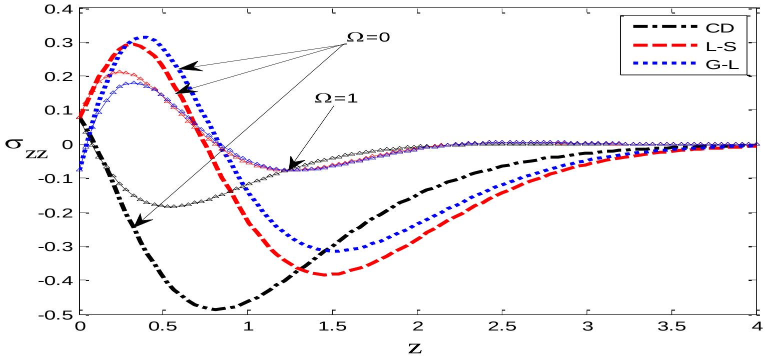

Figure (4). Variation of the stress distribution σ_{zz} in the absence and presence of rotation.

Figure 5 exhibits that the distribution of the stress σ_{xz} begins from the negative value (which is the same point) in the case of $\Omega = 0,1$ in the context of the three theories, and we notice that σ_{xz} satisfies the boundary condition $z = 0$.

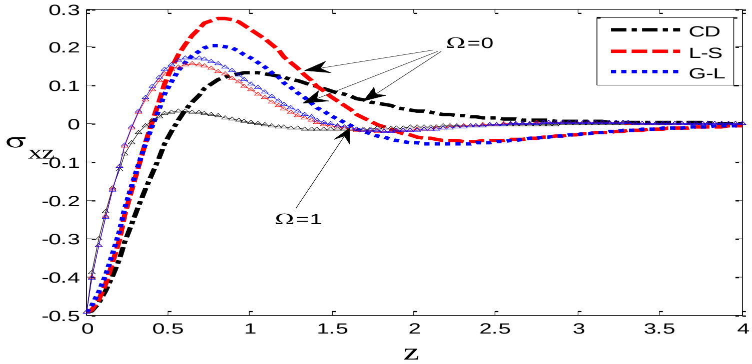

Figure (5). Variation of stress distribution σ_{xz} in the absence and presence of rotation.

Figure 6 explain the distributions of the displacement components u_1 in case of ($\alpha^* = 0.00051, 0$) in the context of the three theories, it noticed that the distribution of u_1 is decreasing with the increase of α^* for $z > 0$.

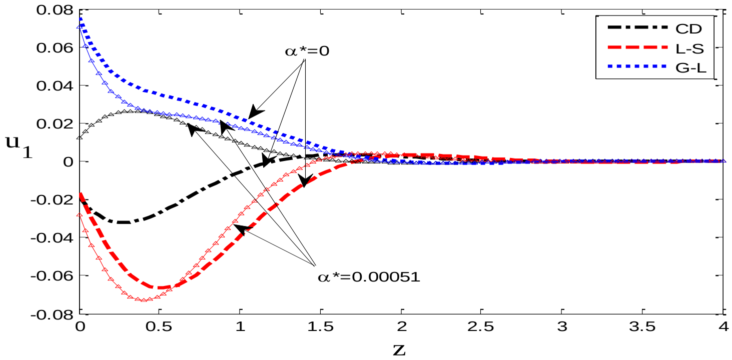

Figure (6). Variation displacement distribution u_1 in the absence and presence of α^*.

Figure 7 shows that the temperature distributes always begins from the positive values and satisfies the boundary conditions at $z = 0$. In the context of the three theories, the values of the temperature T decrease in the range $0 \le z \le 4$. A coincidence of all the curves can be observed when there is a rise in the distance z to reach zero at infinity.

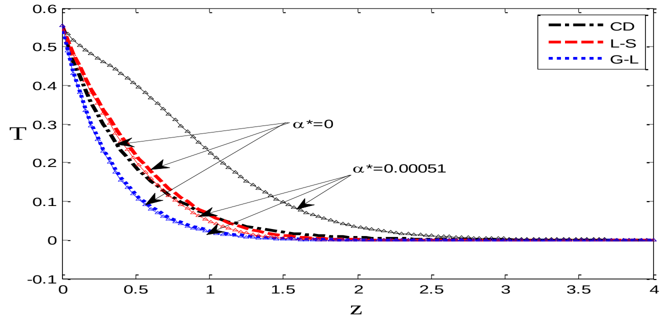

Figure (7). Distribution of the temperature T in the absence and presence of α^*.

Figure 8 depicts the distribution of change in the volume fraction field ϕ in the case of $\alpha^* = 0.00051$, and $\alpha^* = 0$ in the context the three theories, the distribution of ϕ is increasing with the increase of α^* for $z > 0$.

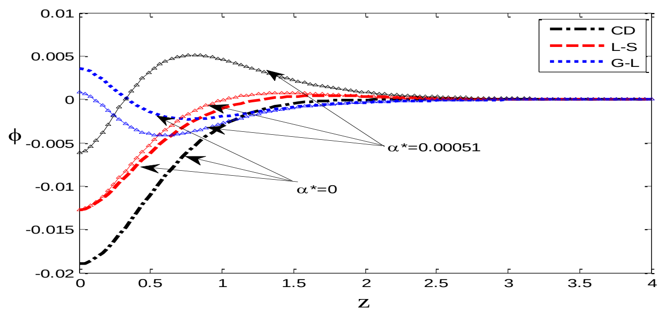

Figure (8). Variation of the volume fraction field ϕ in the absence and presence of α^*.

Figure 9 depicts that the distribution of the stress σ_{zz}, in the case of $(\alpha^* = 0.00051, 0)$ of the (G-L) theory, begins from one positive values and satisfies the boundary conditions at $z = 0$, and The values of distribution σ_{zz} begins from negative values from two theories (CD,LS) and satisfies the boundary conditions at $z = 0$.

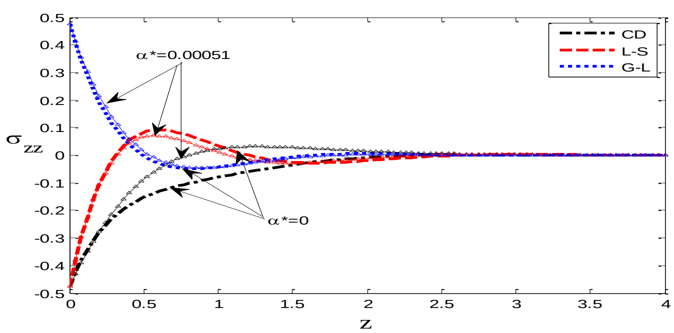

Figure (9). Variation of the stress distribution σ_{zz} in the absence and presence of α^*.

Figure 10 explains the distribution of the stress component σ_{xz} begins from zero and obeys the boundary condition and oscillates like a wave.

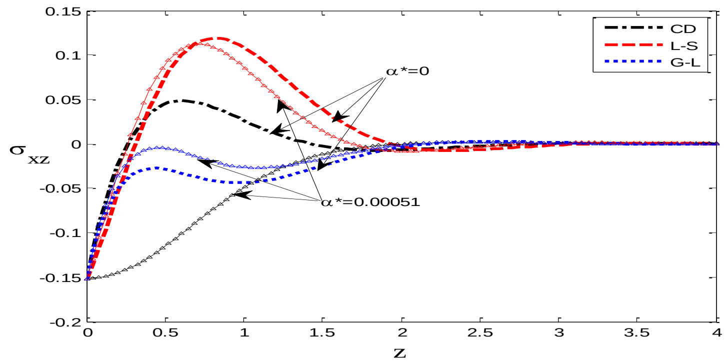

Figure (10). Distribution of the stress σ_{xz} in the absence and presence of α^{*}.

Figure 11 expresses the distribution of the displacement component u_1 in the case of $(\theta = 30, \theta = 15)$ and in the context of the three theories; it noticed that the distribution of u_1 is decreasing with the increase in θ in the range $0 \le z \le 4$ then converges to 0.

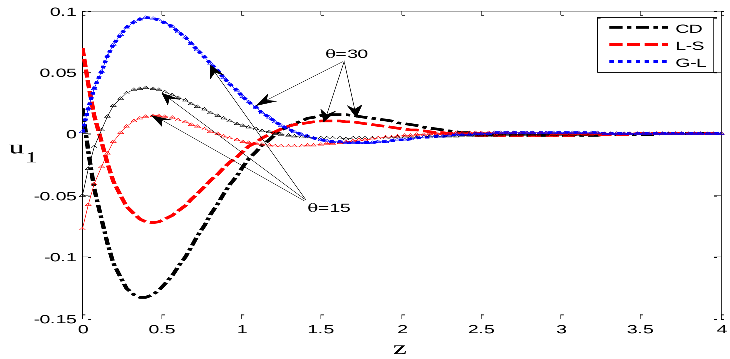

Figure (11). Horizontal displacement distribution u_1 for different values of θ

Figure 12 shows the distribution of the temperature T in the case of $(\theta = 30, \theta = 15)$ and in the context of the three theories, it depicts that different values of angle has a low effect on the temperature.

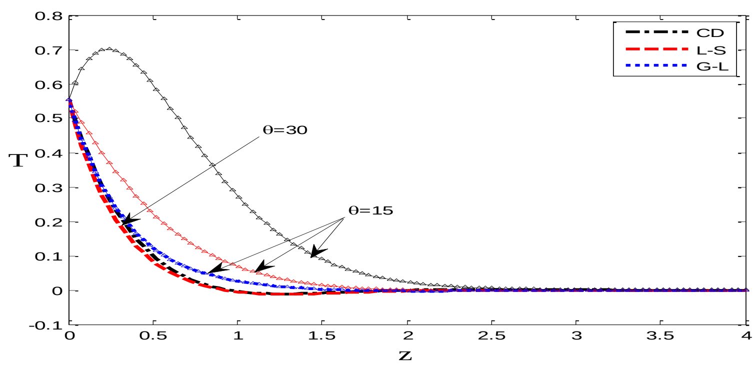

Figure (12). Horizontal displacement distribution u_1 for different values of θ

Figure 13 depicts the distribution of change in the volume fraction field ϕ in the case of $(\theta = 30, \theta = 15)$ and in the context the three theories, the distribution of ϕ is increasing with the increase of θ for $z > 0$.

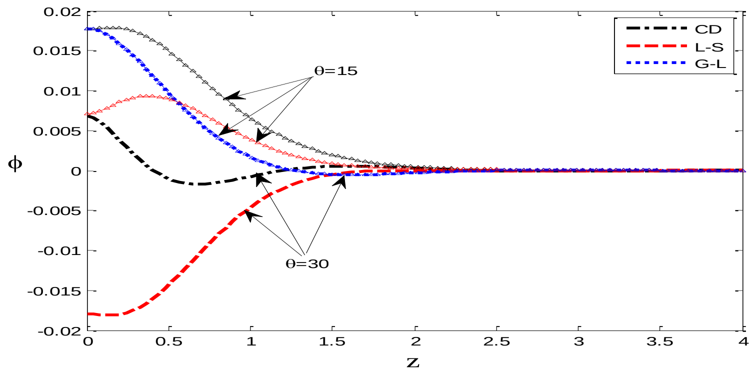

Figure (13). Distribution of the volume fraction field ϕ for different values of θ

Figure 14 and 15 show the distributions of stress components σ_{zz} ,and σ_{xz} , in the case of $(\theta = 30, \theta = 15)$ and in the context of the three theories. It noticed that the distribution of stress components increases with the increase of θ , while, the distribution of stress component σ_{xz} , decreases with the increase of θ .

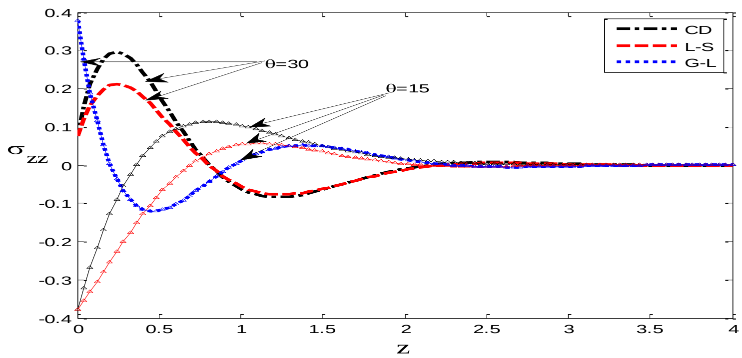

Figure (14). Distribution of the stress component σ_{zz} for different values of θ

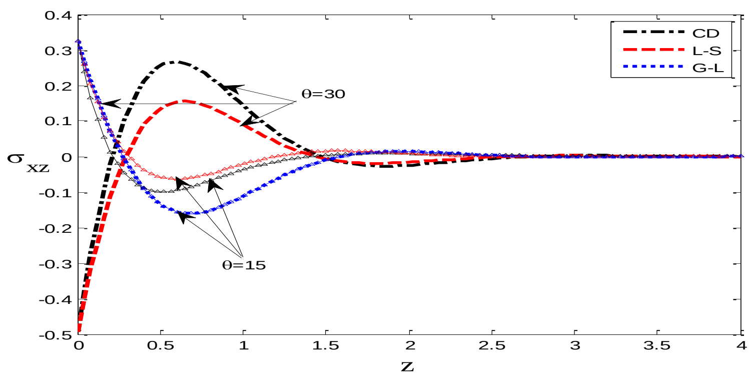

Figure (15). Distribution of the stress σ_{xz} for different values of θ

3D curves are representing the complete relations between physical quantities and both factors of distance as shown in Figure ure s 16-18, in the presence, temperature dependent $\alpha^{*}=0.00051$, of rotation $\Omega=1$ and Inclined load dependent $\theta=30$ in generalized thermoelastic medium with pours Material in the context of the (G-L) theory. These Figure ure s are very important to study the dependence of these physical quantities The curves obtained are highly dependent on the distance from origin, and all the physical quantities are moving in the wave propagation.

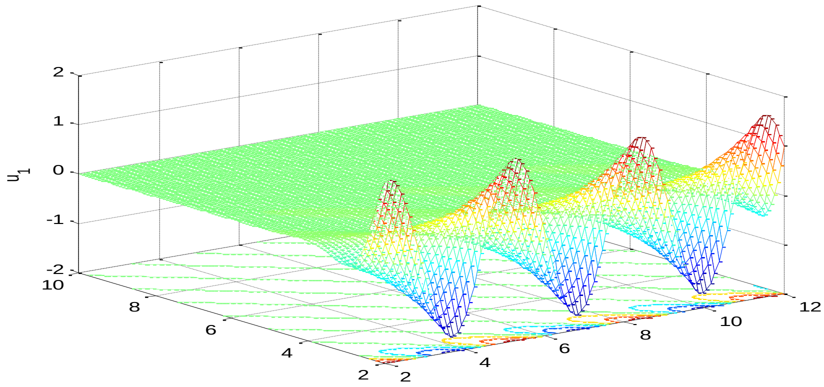

Figure (16). 3D variation of the displacement u_1 with the variation of x , z .

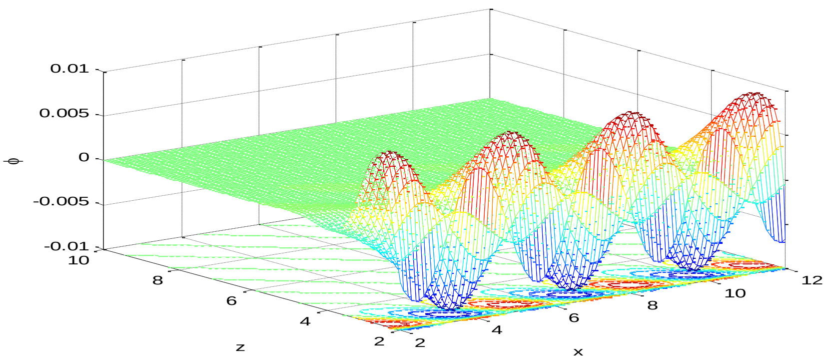

Figure (17). (3D) variation of the displacement of the displacement component ϕ with the variation of x , z

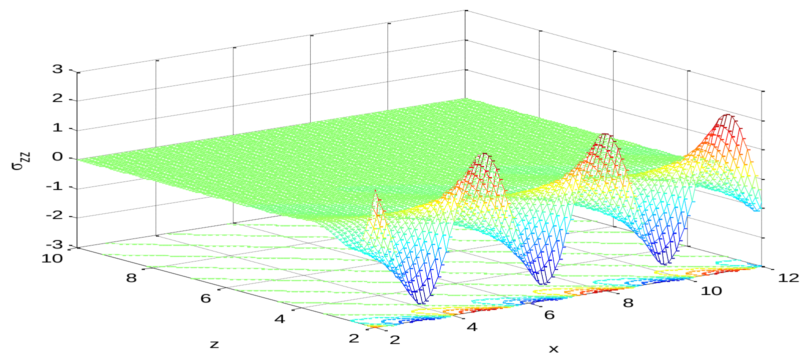

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Figure (18). (3D) variation of the stress distribution σ_{zz} with the variation of x , z .

CONCLUSION

We can conclude that the curves of the physical quantities with (CD) in most of the Figure ure s are lower in comparison with those under (L-S) theory and (G-L) theory due to the relaxation times. The values of all the physical quantities converge to 0 by increasing the distance z and all the functions are continuous. . The rotation and the inclined load play a significant role in the distribution of all the physical quantities.

Duality of interest: The authors declare that they have no duality of interest associated with this manuscript.

Funding: No specific funding was received for this work.

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