Al-Mukhtar Journal of Basic Sciences 00 (0): 01-08, 2025
Doi: https://doi.org/10.54172/bjhz9304

Research Article **Open Access**

Solving Fuzzy Ordinary Differential Equations Using Homotopy Analysis Method

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Received:
29 May 2025

Accepted:
26 October 2025

Publish online:
04 November 2025

Abstract

Fuzzy ordinary differential equations (FODEs) extend classical ordinary differential equations (ODEs) to systems characterized by uncertainty and imprecision, often modeled using fuzzy set theory. In this work we use homotopy analysis method (HAM) to solve FODEs, and show how HAM works. (HAM) is an easy and effective method to solve linear and nonlinear ordinary differential equations. HAM does not need small parameters or special tricks like linearization. Instead, it builds a smooth path from a simple starting guess to the real solution by using a special parameter and function.

Keywords: Fuzzy Concepts, Fuzzy Ordinary Equations, Homotopy Analysis Method**.**

INTRODUCTION

Fuzzy ordinary differential equations (FODEs) generalize classical ordinary differential equations (ODEs) to better model systems with uncertainty and imprecision, using the framework of fuzzy set theory. In contrast to traditional ODEs where initial conditions and parameters are precisely defined. FODEs use fuzzy numbers and functions to deal with uncertainty in real-world problems. This method is especially useful in engineering, biology, economics, and decision-making. where uncertainty cannot be ignored. Various methods, including generalized differentiability concepts, have been developed to define and solve FODEs. Analytical and numerical techniques are employed to obtain fuzzy-valued solutions, and stability analysis is adapted to the fuzzy context. Recent researches have focused on improving solution accuracy, developing efficient computational algorithms, and extending fuzzy differential models to fractional and stochastic domains. This paper presents one of the effective approaches for solving FODEs which is HAM.

The basic idea of HAM involves embedding the original fuzzy problem into a family of simpler problems controlled by an embedding parameter. An initial approximation is first assumed, and through the homotopy, a series of linear sub-problems is generated. These sub-problems are solved sequentially to build a convergent series solution. One of the strengths of HAM lies in its auxiliary parameter, which allows control over the convergence of the solution, ensuring both stability and accuracy.
Ghanbari (2012) used two methods for solving first order linear fuzzy differential equations.

These methods are variational iteration method (VIM) and Adomian decomposition method (ADM), and compared between the two methods.

(Jameel et all 2014) used the Fuzzy homotopy analysis method to obtain the approximate analytical solutions of the high order ordinary fuzzy equations. (Jameel 2015) applied the homotopy perturbation method to develop and find an approximate-analytical solution of fuzzy initial value problems involving a nonlinear first order ordinary differential equation.

In 2020, Ali and Ibraheem developed some fuzzy analytical and numerical solutions of the linear first order fuzzy initial value problems by using homotopy analysis method based on some of the Approximate method. In this work we show how we can solve FODE using homotopy analysis method.

Some Basic Features of The Fuzzy Concepts

Definition (**1**) (Fuzzy Set) [Shokri 2007] (Gasilov et. all 2012)
Let Y be a classical set of objects, called the universal set, whose generic elements are denoted by y. The membership in a classical subject A of Y is often viewed as a characteristic function μ_A from y onto {0,1}, such that:

$$\mu_A(y) = \begin{cases} 1 \ if \ y \in A, \\ 0 \ if \ y \notin A, \end{cases}$$

The set {0,1} is called a valuation set. If the valuation set is allowed to be real interval [0,1], then A is called a fuzzy sets (which is denoted in this case by $\tilde{A}$) and $\mu_{\tilde{A}}(y)$ is the grade of membership of y in $\tilde{A}$. Also, it is remarkable that the closer the value of $\mu_{\tilde{A}}(y)$ to 1, the more y belong to $\tilde{A}$. $\tilde{A}$ is a subset of Y that has no sharp boundary. The fuzzy set $\tilde{A}$ is a completely characterized by the set of pairs:

$$\tilde{A} = \{(y, \mu_{\tilde{A}}(y)) : y \in Y, 0 \le \mu_{\tilde{A}}(y) \le 1\}$$

Definition (**2**) (α- Level Set) (Gasilov et. all 2012)
The α - level (or α - cut) set of a fuzzy set $\tilde{A}$ labeled by A_α, is the crisp set of all y in Y such that: $\mu_{\tilde{A}}(y) \ge \alpha$; i.e.

$$A_\alpha = \{y \in Y : \mu_{\tilde{A}}(y) \ge \alpha, \alpha \in [0,1]\}$$

Definition (**3**) (Fuzzy Number) [Jameel 2015] [Nadeem 2023]
A fuzzy number is a generalization of a regular real number in the sense that it does not refer to one single value but to a connected set of possible values, where each possible value has its own weight between 0 and 1.
By other words a fuzzy number $\tilde{u}$ is completely determined by an ordered pair of functions $(\bar{u}(\alpha), \bar{u}(\alpha)), 0 \le \alpha \le 1,$ which satisfy the following conditions:
1) $\underline{u}\ (\alpha)$ is a bounded left continuous and increasing function over [0,1].
2) $\bar{u}(\alpha)$ is a bounded wright continuous and decreasing function over [0,1].
3) $\bar{u}(\alpha) \le \bar{u}(\alpha), 0 \le \alpha \le 1$

Remark (1)
u is named a crisp number if:
$\bar{u}(\alpha) = \underline{u}(\alpha) = u \ \ 0 \le \alpha \le 1.$
All the set of the fuzzy numbers is characterized by E.

Definition (4) (Fuzzy Function) [Jameel 2015]
A mapping $f: T \to E$ for some interval $T \subseteq E$ is named a fuzzy function or fuzzy process with non-fuzzy variable (crisp variable), and we denote α - level sets by

$$[f(t)]_\alpha = \left[\underline{f}(t;\alpha), \overline{f}(t;\alpha)\right]$$

Where $t \in T$, $\alpha \in [0,1]$. That is to say, the fuzzifying function is a mapping from a domain to a fuzzy set of range. Fuzzifying function and the fuzzy relation coincide with each other in the mathematical way. We refer to $\underline{f}$ and $\overline{f}$ as the lower and upper branches on f.

Definition (5) [Chalco-Cano 2020]
If $f: T \to E$ is a fuzzy function, then the gH-derivative of a fuzzy function at $t_0 \in T$ is defined as

$$f'(t_0) = \lim_{h \to 0} \frac{1}{h}\left[f(t_0 + h) -_{gH} f(t_0)\right]$$

If $f'(t_0) \in E$ satisfying the above equation then f is gH-differentiable at t_0

Definition (6) (FODE)
A fuzzy ordinary differential equation (FODE) of the first order is an equation of the form

$$\frac{dy}{dt} = f\big(t, y(t)\big), t \in [t_0, T]$$

where $y(t)$ is an unknown fuzzy-valued function, and f is a given fuzzy-valued function.
In addition, an initial condition is often specified: $y(t) = y_0$, where y_0 is a fuzzy initial condition at time t_0.

Fuzzy Homotopy Analysis Method
Fuzzy homotopy analysis method is a type of the fuzzy semi- analytical methods used to find the fuzzy series solution (fuzzy semi-analytical solution) of the FIVBs. Through the use of homotopy, this method reformulates fuzzy nonlinear equations into a fuzzy series composed of linear equations that converges. Essentially, the approach focuses on producing a fuzzy series of approximate solutions that closely represent the exact analytical solution of the fuzzy initial value problems. The fundamental mathematical concepts of the fuzzy homotopy analysis method are similar to the fundamental mathematical concepts of the homotopy analysis method, but with the use of the concepts of fuzzy theory. This means that, solving any FIVB by using fuzzy homotopy analysis method is depend on transform the FIVB into a system of non-fuzzy initial value problems by applying special steps. and then using the homotopy analysis method to solve this system. The fuzzy homotopy analysis method provides us with both the freedom to choose proper base fuzzy functions for approximating a non-linear fuzzy problem and a simple way to ensure the convergence of the fuzzy series solution.
To illustrate how the HAM works, we begin with the following differential equation

$$N[y(t)]_\alpha = 0$$

Where N is a fuzzy non-linear operator, t is an independent crisp variable, and y(t) is an unknown fuzzy function.
Let we consider the zero-order fuzzy deformation equation

$$(1-p)L[\emptyset(t;p) - y_0(t)] = phH(t)N[\emptyset(t;p)] \qquad (1)$$

Where $p \in [0,1]$, is the homotopy embedding parameter, $(h \neq 0)$ is the convergence control parameter , L is the fuzzy linear operator, $y_0(t)$ is the fuzzy initial guess of y(t) and $\emptyset(t;p)$ is a fuzzy function. $H(t) \neq 0$,
We can see that, when $p = 0$ we get;

$$[\emptyset(t;0)]_\alpha = [y_0(t)]_\alpha \tag{2}$$

And when $p = 1$

$$[\emptyset(t;1)]_\alpha = [y(t)]_\alpha \tag{3}$$

From Eq. (2) and (3) it's clearly that, when *p* is increasing from 0 to 1, the fuzzy solutions $[\underline{\emptyset}(t;p)]_\alpha$ and $[\overline{\emptyset}(t;p)]_\alpha$ varies from the fuzzy initial guess $[y_0(t)]_\alpha$ to the fuzzy solution $[y(t)]_\alpha$
.
Now By expanding $[\emptyset(t;p)]_\alpha$ in Taylor series with respect to *p*, get the following

$$[\emptyset(t,p)]_\alpha = [y_0(t)]_\alpha + \sum_{m=1}^{\infty} [y_m(t)p^m]_\alpha \tag{4}$$

Where $[y_m(t)]_\alpha = \frac{1}{m!} \times \frac{\partial^m [\emptyset(t;p)]_\alpha}{\partial p^m} |_{p=0}$

Now, if we suitably determined the followings: the fuzzy linear operator, the fuzzy initial guess, the auxiliary parameter *h*, and the auxiliary fuzzy function, then the fuzzy Eq. (4) converges at p = 1, and we have:

$$[\emptyset(t;1)]_\alpha = [y(t)]_\alpha = [y_0(t)]_\alpha + \sum_{m=1}^{\infty} [y_m(t)]_\alpha$$

We define the fuzzy vectors $[\vec{y_n}(t)]_\alpha = \{[y_0(t)]_\alpha, [y_1(t)]_\alpha, \dots, [y_n(t)]_\alpha\}$
by differentiating Eq. (1) m- times respect to the *p*, and then setting *p = 0* and dividing them by m! we obtain the fuzzy deformation equation of order m:

$$L[y_m(t;p) - \chi_m y_{m-1}(t)] = hH(t)R_m[\vec{y}_{m-1}] \tag{5}$$

$$R_m[\vec{y}_{m-1}]_\alpha = \frac{1}{(m-1)!} \times \frac{\partial^{m-1} N[\emptyset(t;p)]_\alpha}{\partial p^{m-1}}, \tag{6}$$

$$\chi_m = \begin{cases} 0, m \leq 1 \\ 1, m > 1 \end{cases}$$

Applying L^{-1} on Eq. (5) we get

$$[y_m(t;p) - \chi_m y_{m-1}(t)] = hL^{-1}[H(t)R_m[\vec{y}_{m-1}]],$$

$$[y_m(t)]_\alpha = \chi_m y_{m-1}(t)] + hL^{-1}[H(t)R_m[\vec{y}_{m-1}]_\alpha]. \tag{7}$$

Then we can obtain $[y_m(t)]_\alpha, for\ all\ m \geq 1$

$$y(t) = \sum_{m=1}^{M} [y_m(t)]_\alpha, \tag{8}$$

as we have

$$\underline{y}(t;\alpha) = \sum_{m=1}^{M} [\underline{y}_m(t)]_\alpha \tag{9}$$

$$\overline{y}(t;\alpha) = \sum_{m=1}^{M} [\overline{y}_m(t)]_\alpha \tag{10}$$

Example: find solution of the following nonlinear fuzzy initial value equation

$$y'(t) - y^2(t) = 1, \tag{11}$$

$$[y(0)]_\alpha = [0.1\alpha - 0.1, 0.1 - 0.1\alpha], \qquad \alpha \in [0,1], \qquad t \in [0,0.2].$$

Where the exact solution is

$$\left[\underline{\boldsymbol{y}}(\boldsymbol{t})\right]_\alpha = \mathbf{tan}(\boldsymbol{t} + \boldsymbol{tan}^{-1}(\mathbf{0.1}\boldsymbol{\alpha} - 0.1))$$

$$[\overline{\boldsymbol{y}}(\boldsymbol{t})]_\alpha = \mathbf{tan}(\boldsymbol{t} + \boldsymbol{tan}^{-1}(\mathbf{0.1} - 0.1\alpha)).$$

Solution: The fuzzy linear operator is:

$$L[\emptyset(t;p)]_\alpha = \left[L\left(\underline{\emptyset}(t;p)\right), L\left(\overline{\emptyset}(t;p)\right)\right]_\alpha,$$

and the nonlinear operator

$$N[\emptyset(t;p)]_\alpha = \left[N\left(\underline{\emptyset}(t;p)\right), N\left(\overline{\emptyset}(t;p)\right)\right]_\alpha,$$

the linear operator defined as following:

$$L\left[\underline{\emptyset}(t;p)\right]_\alpha = \left[\frac{\partial \underline{\emptyset}(t;p)}{\partial t}\right]_\alpha, \qquad L\left[\overline{\emptyset}(t;p)\right]_\alpha = \left[\frac{\partial \overline{\emptyset}(t;p)}{\partial t}\right]_\alpha.$$

Wright the Eq. (11) as following

$$\overline{y'}(t;\alpha) = \overline{y}^2(t;\alpha) + 1,$$

$$\underline{y}'(t;\alpha) = \underline{y}^2(t;\alpha) + 1,$$

with the initial value problem

$$\overline{y'}(0;\alpha) = 0.1\alpha - 0.1,$$

$$\underline{y}'(0;\alpha) = 0.1 - 0.1\alpha,$$

And the linear operator

$$\underline{L} = \frac{\partial[\underline{\emptyset}(t;p)]_\alpha}{\partial t},$$

$$\overline{L} = \frac{\partial[\overline{\emptyset}(t;p)]_\alpha}{\partial t},$$

$$[\underline{y}_m(t)]_\alpha = \chi_m \underline{y}_{m-1}(t) + h\underline{L}^{-1}\left[H(t)R_m\left[\underline{\vec{y}}(t)_{m-1}\right]_\alpha\right]$$

$$[\overline{y}_m(t)]_\alpha = \chi_m \overline{y}_{m-1}(t) + h\overline{L}^{-1}\left[H(t)R_m\left[\overline{\vec{y}}_{m-1}\right]_\alpha\right]$$

$$\overline{y}_m(0;\alpha) = 0, \underline{y}_m(0;\alpha) = 0.$$

$$R_m\left[\overline{\vec{y}}_{m-1}\right]_\alpha = \overline{y'}_{m-1}(t,\alpha) - \overline{y}^2{}_{m-1} - 1$$

$$R_m\left[\underline{\vec{y}}_{m-1}\right]_\alpha = \underline{y}'_{m-1}(t,\alpha) - \underline{y}^2{}_{m-1} - 1$$

$$R_1\left[\overline{\vec{y}}_{m-1}\right]_\alpha = \overline{y'}_0(t,\alpha) - \overline{y}^2{}_0 - 1$$

$$R_1\left[\underline{\vec{y}}_{m-1}\right]_\alpha = \underline{y}'_1(t,\alpha) - \underline{y}^2{}_1 - 1$$

$$R_1\left[\overline{\vec{y}}_{m-1}\right]_\alpha = -0.01\alpha^2 + 0.02\alpha - 1.01$$

$$R_1\left[\underline{\vec{y}}_{m-1}\right]_\alpha = -0.01\alpha^2 + 0.02\alpha - 1.01$$

$$[\overline{y}_1]_\alpha = \int(-0.01h\alpha^2 + 0.02h\alpha - 1.01h)dt$$

$$[\overline{y}_1]_\alpha = (-0.01\alpha^2 + 0.02\alpha - 1.01)ht$$

By the same

$$\left[\underline{y}_1\right]_\alpha = (-0.01h\alpha^2 + 0.02h\alpha - 1.01h)t$$

Now, to find $[y_2(t)]_\alpha$, we have from Eq. (4)

$$\left[\underline{\emptyset}(t;p)\right]_\alpha = \left[\underline{y}_0(t;\alpha)\right] + p[\underline{y}_1(t;\alpha)] + p^2[\underline{y}_2(t;\alpha)]$$

$$\left[\overline{\emptyset}(t;p)\right]_\alpha = [\overline{y}_0(t;\alpha)] + p[\overline{y}_1(t;\alpha)] + p^2[\overline{y}_2(t;\alpha)]$$

$$L\left(\overline{y}_2(t;\alpha) - \overline{y}_1(t;\alpha)\right) = h\overline{R}_2(t;\alpha)$$

$$L\left(\underline{y}_2(t;\alpha) - \underline{y}_1(t;\alpha)\right) = h\underline{R}_2(t;\alpha)$$

From eq. (6) we have

$$\overline{R}_2(t;\alpha) = \frac{\partial\left[N(\overline{\emptyset}(t;p))_\alpha\right]}{\partial p}\Big|_{p=0}$$

$$\underline{R}_2(t;\alpha) = \frac{\partial\left[N(\underline{\emptyset}(t;p))_\alpha\right]}{\partial p}\Big|_{p=0}$$

Apply the following steps

$$\frac{\partial\left[\underline{\emptyset}(t;p)\right]_\alpha}{\partial t} = \frac{\partial\left[\underline{y}_0(t)\right]_\alpha}{\partial t} + p\frac{\partial\left[\underline{y}_1(t)\right]_\alpha}{\partial t} + p^2\frac{\partial\left[\underline{y}_2(t)\right]_\alpha}{\partial t}$$

$$\frac{\partial\left[\overline{\emptyset}(t;p)\right]_\alpha}{\partial t} = \frac{\partial[\overline{y}_0(t)]_\alpha}{\partial t} + p\frac{\partial[\overline{y}_1(t)]_\alpha}{\partial t} + p^2\frac{\partial[\overline{y}_2(t)]_\alpha}{\partial t}$$

$$N\left(\underline{\emptyset}(y;p)\right)_\alpha = \frac{\partial\left[\underline{y}_0(t)\right]_\alpha}{\partial t} + p\frac{\partial\left[\underline{y}_1(t)\right]_\alpha}{\partial t} + p^2\frac{\partial\left[\underline{y}_2(t)\right]_\alpha}{\partial t} - \left(\left[\underline{y}_0(t) + p\underline{y}_1(t) + p^2\underline{y}_2(t)\right]_\alpha\right)^2 - 1$$

$$N\left(\overline{\emptyset}(y;p)\right)_\alpha = \frac{\partial[\overline{y}_0(t)]_\alpha}{\partial t} + p\frac{\partial[\overline{y}_1(t)]_\alpha}{\partial t} + p^2\frac{\partial[\overline{y}_2(t)]_\alpha}{\partial t} - \left([\overline{y}_0(t) + p\overline{y}_1(t) + p^2\overline{y}_2(t)]_\alpha\right)^2 - 1$$

$$\frac{\partial\left[N\left(\underline{\emptyset}(t;p)\right)\right]_\alpha}{\partial p} = \frac{\partial\left[\underline{y}_1(t)\right]_\alpha}{\partial t} + 2p\frac{\partial\left[\underline{y}_2(t)\right]_\alpha}{\partial t} - 2\left[\left(\underline{y}_0(t) + p\underline{y}_1(t) + p^2\underline{y}_2(t)\right)_\alpha\right]\left(\underline{y}_1(t) + 2p\underline{y}_2(t)\right)$$

$$\frac{\partial\left[N\left(\overline{\emptyset}(t;p)\right)\right]_\alpha}{\partial p} = \frac{\partial[\overline{y}_1(t)]_\alpha}{\partial t} + 2p\frac{\partial[\overline{y}_2(t)]_\alpha}{\partial t} - 2\left[\left(\overline{y}_0(t) + p\overline{y}_1(t) + p^2\overline{y}_2(t)\right)_\alpha\right]\left(\overline{y}_1(t) + 2p\overline{y}_2(t)\right)$$

$$\left[\underline{R}_2\right]_\alpha = \frac{\partial\left[\underline{y}_1(t)\right]_\alpha}{\partial t} - 2\left[\underline{y}_0(t)\underline{y}_1(t)\right]_\alpha$$

$$\left[\overline{R}_2\right]_\alpha = \frac{\partial[\overline{y}_1(t)]_\alpha}{\partial t} - 2[\overline{y}_0(t)\overline{y}_1(t)]_\alpha$$

$$\left[\underline{R}_2\right]_\alpha = (-0.01\alpha^2 + 0.02\alpha - 1.01)h + (0.002\alpha^3 - 0.006\alpha^2 + 0.206\alpha - 0.202)ht$$

$$\left[\overline{R}_2\right]_\alpha = (-0.01\alpha^2 + 0.02\alpha - 1.01)h + (0.002\alpha^3 + 0.006\alpha^2 - 0.206\alpha + 0.202)ht$$

$$L\left[\underline{y}_2(t)-\underline{y}_1(t)\right]_\alpha = (-0.01\alpha^2+0.02\alpha-1.01+0.002\alpha^3t-0.006\alpha^2t+0.206\alpha t-0.202t)h^2$$

And

$$L\left[\overline{y}_2(t)-\overline{y}_1(t)\right]_\alpha = (-0.01\alpha^2+0.02\alpha-1.01-0.002\alpha^3t+0.006\alpha^2t-0.206\alpha t+0.202t)h^2$$

Applying the L^{-1} operator on the last two equations

$$\left[\underline{y}_2(t)-\underline{y}_1(t)\right]_\alpha = (-0.01\alpha^2t+0.02\alpha t-1.01t+0.001\alpha^3t^2-0.003\alpha^2t^2+0.103\alpha t^2-0.101t^2)h^2$$

$$\left[\overline{y}_2(t)-\overline{y}_1(t)\right]_\alpha = (-0.01\alpha^2t+0.02\alpha t-1.01t-0.001\alpha^3t^2+0.003\alpha^2t^2-0.103\alpha t^2+0.101t^2)h^2$$

So,

$$\left[\underline{y}_2(t)\right]_\alpha = (-0.01\alpha^2t+0.02\alpha t-1.01t+0.001\alpha^3t^2-0.003\alpha^2t^2+0.103\alpha t^2-0.101t^2)h^2 + (-0.1\alpha^2+0.02\alpha-1.01)th$$

$$\left[\overline{y}_2(t)\right]_\alpha = (-0.01\alpha^2t+0.02\alpha t-1.01t-0.001\alpha^3t^2+0.003\alpha^2t^2-0.103\alpha t^2+0.101t^2)h^2 + (-0.01\alpha^2+0.02\alpha-1.01\alpha)th$$

By the same way we can get the other component**s.**

the fuzzy solution is;

$$[y(t)]_\alpha = \left[\left[\underline{y}(t)\right]_\alpha, [\overline{y}(t)]_\alpha\right]$$

$$\left[\underline{y}(t)\right]_\alpha = 0.1-\ 0.1\alpha+(-0.01h\alpha^2+0.02h\alpha-1.01h)t + (-0.01\alpha^2t+0.02\alpha t-1.01t+0.001\alpha^3t^2-0.003\alpha^2t^2+0.103\alpha t^2-0.101t^2)h^2+(-0.1\alpha^2+0.02\alpha-1.01)th$$

$$[\overline{y}(t)]_\alpha = 0.1\alpha-0.1+(-0.01\alpha^2+0.02\alpha-1.01)ht + (-0.01\alpha^2t+0.02\alpha t-1.01t-0.001\alpha^3t^2+0.003\alpha^2t^2-0.103\alpha t^2+0.101t^2)h^2+(-0.01\alpha^2+0.02\alpha-1.01\alpha)th$$

For the best final solution let $h=-1$,

$$\left[\underline{y}(t)\right]_\alpha = 0.1-\ 0.1\alpha-(-0.01\alpha^2+0.02\alpha-1.01)t + (-0.01\alpha^2t+0.02\alpha t-1.01t+0.001\alpha^3t^2-0.003\alpha^2t^2+0.103\alpha t^2-0.101t^2)-(-0.1\alpha^2+0.02\alpha-1.01)t$$

$$[\overline{y}(t)]_\alpha = 0.1\alpha-0.1-(-0.01\alpha^2+0.02\alpha-1.01)t + (-0.01\alpha^2t+0.02\alpha t-1.01t-0.001\alpha^3t^2+0.003\alpha^2t^2-0.103\alpha t^2+0.101t^2)-(-0.01\alpha^2+0.02\alpha-1.01\alpha)t$$

To illustrate the accuracy of the results, we take the following values $\boldsymbol{t}=\boldsymbol{0}, \alpha=0.2$ and we get

$$[\boldsymbol{y}(\boldsymbol{0})]_{\boldsymbol{0.2}} = [-0.08,0.08]$$

the exact solution is

$$[\boldsymbol{y}(\boldsymbol{0})]_{\boldsymbol{0.2}} = [-0.079999988,0.079999988]$$

If $t=0.2, \alpha=0.2$,

$$[\boldsymbol{y}(\boldsymbol{0.2})]_{\boldsymbol{0.2}} = [0.120794700,0.287235770]$$

the exact solution is

$$[\boldsymbol{y}(\boldsymbol{0.2})]_{\boldsymbol{0.2}} = [0.120751827,0.287370261]$$

RESULTS

In this study, we successfully demonstrated the application of the homotopy analysis method (HAM) for solving fuzzy ordinary differential equations (FODEs), which naturally extend classical ODEs to account for uncertainty and imprecision. Our results show that HAM is a highly effective and flexible tool, offering advantages over traditional methods by eliminating the need for small parameters or linearization techniques. HAM could be applying for linear and non-linear equations also homogenous and non-homogenous equations.

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