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Some Approximation Spaces via Supra Topology

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Abstract

This paper introduces a new space based on a generalized neighbourhood system using the concept of a supra topology, termed a supra approximation space (briefly S^nAS). We investigate several properties of S^nAS and compare its advantages with those of the classical neighbourhood approximation space. Furthermore, we define and study a novel class of separation axioms using S^n-open in a supra approximation space (S^nAS). Finally, we explore some of their properties in the context of information system, particularly in approximation process of approximation and definability.

Keywords: Neighbourhood System; Rough Set Theory; Approximation Space; Supra Topology; Topological Space; T_0 ; T_1 And T_2.

INTRODUCTION

In 1983 (Mashhour, 1983) introduced supra topological spaces and studied S-continuous functions and S^*-continuous functions. They introduced the notion of S -open and S-closed sets and characterized these sets using S -closure and S-interior operators respectively.

Rough set theory, proposed by Pawlak in 1998 (Pawlak & Systems, 1998) extends classical set theory. Pawlak introduced the notion of an approximation space (U, R), where U is a universe set and R is an equivalence relation. Within this framework, he defined the lower approximation ($\underline{X}$), upper approximation ($\overline{X}$), and the boundary region ($bnd(X)$) of any subset $X \subseteq U$. A fundamental result establishes the relationship between these concepts: $(\underline{X}) \subseteq X \subseteq (\overline{X})$ and $bnd(X) = (\overline{X}) - (\underline{X})$.

In (Császár, 2004), the author further explored separation axioms using generalized topology notions. This paper introduces a novel structure called a supra approximation space (briefly S^nAS). We investigate key definitions and properties of approximations within S^n***AS***, including the formulation of lower and upper approximations. Moreover, we define and analyze new separation axioms based S^n-open sets, examining their fundamental properties. This work lays the foundation for future applications of separation axioms in related fields.

Definition 1.1. (Lashin et al., 2005)
consider (U, R) is a generalized approximation space. Then:

1. $N(x) = \{y \in U: xRy\}$ is called the right neighborhood of an element x,
2. $N(U) = \{N(x): x \in U\}$ is the collection of all neighborhoods in (U, R).

2- A Supra Approximation Space
We use a neighborhood to construct a supra approximation space ($S^n AS$). This space relies on a supra topological structure. Additionally, we investigate some definitions, including:

1. supra lower approximation,
2. supra upper approximation,
3. $S^n -$ undefinable (rough) set in $S^n AS$,
4. S^n-internally, S^n-externally, and S^n-totally definable sets.

Definition 2.1.
consider (U, R) is a generalized approximation space. We define the class of a supra on (U, R) as:
$S^n(U) = \{\phi, U, \{X \subseteq U: X = \cup_{x \in U} N(x)\}\}$,
where $N(x)$ is the right neighborhood of x for all $x \in U$.

1. The members of the supra set $S^n(U)$ are called S^n-open sets,
2. The pair (U, R, S^n) is called a supra approximation space (in short, S^nAS),
3. The complement of an S^n-open set is called an S^n-closed set,
4. The class of all S^n-closed sets is denoted by $\left(S^n(U)\right)^c$.

Example2.1.
Let (U, R, S^n) be an supra approximation space $S^n AS$, where $U = \{a, b, c, d\}$, R be a binary relation on U
$R = \{(a, b), (a, d), (b, b), (b, c), (c, d), (d, a)\}$.
The neighborhoods are defined as:
$N(a) = \{b, d\}$,
$N(b) = \{b, c\}$,
$N(c) = \{d\}$,
$N(d) = \{a\}$.
The supra set is:

$S^n(U) = \{\emptyset, \{a\}, \{d\}, \{b, c\}, \{b, d\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U\}$,
and the class of S^n-closed sets is:
$\left(S^n(U)\right)^c = \{\emptyset, \{a\}, \{c\}, \{d\}, \{b, c\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{b, c, d\}, U\}$.

Definition 2.2.
consider (U, R, S^n) is $S^n AS$, $X \subseteq U$. Then, X is called S^n-definable (or exact) if $S^n(X) = \left(S^n(X)\right)^c$.

Definition 2.3.
Let U be a nonempty set and $N(U)$ be a nonempty collection of subsets of U. $N(U)$ is called supra topology (briefly$S^n T$) on U if:

1. $\emptyset, U \in N(U)$, and
2. For any collection $\{N(x)\}_{x\in U} \subseteq N(U)$,the union $\cup_{x\in U} N(x) \in N(U)$ where $N(x)$ is the right neighborhood of x for all $x \in U$.

The pair$\big(X, N(U)\big)$ is called a supra topological space (briefly S^nTS).The members of $N(U)$ are called supra- open sets, and a subset of a supra topological space $\big(U, N(U)\big)$is called supra-closed set if its complement is a supra- open set.

Definition 2.4.
Consider (U, R, S^n) is S^nAS, $X \subseteq U$. We define:

1. The supra lower approximation of X(in short $\underline{S^n}(X)$) as:

$$\underline{S^n}(X) = \cup\{G : G \in S^n(X),\ G \subseteq X\};$$

2. The supra upper approximation of X (in short $\overline{S^n}(X)$) as:

$$\overline{S^n}(X) = \cap\left\{F : F \in \big(S^n(X)\big)^c,\ X \subseteq F\right\}.$$

Theorem 2.1.
Let (U, R, S^n)be S^nAS on U and $B \subseteq U$.Then, $x \in \overline{S^n}(B)$ if and only if $G \cap B \neq \emptyset$ for every S^n-open set G such that $x \in G$.

Proof:
This follows directly from Definition 2.4.

Definition2.5.
Let (U, R, S^n) be an S^nAS. We say that the union of any family of elements of $S^n(U)$ is in $S^n(U)$.

Theorem 2.2.
Let $(U, R. S^n)$ be an S^nAS on U, and let $X \subseteq U$. Then:

1. $\underline{S^n}(X)$ is a supra set.
2. $\underline{S^n}(X)$ is the largest supra set contained in X.
3. X is a supra set if and only if $\underline{S^n}(X) = X$.

Proof:
This follows from the definitions of S^n. set and $\underline{S^n}(X)$.

Definition 2.6.
Let (U, R, S^n) be an S^nAS, and let $X \subseteq U$.Then:

1. The supra boundary of X (denoted byS^n- $bnd(\mathrm{X})$) is:

   $S^n\text{-}bnd(\mathrm{X}) = \overline{S^n}(X) - \underline{S^n}(X).$

2. The supra internal edge of X (denoted by $\underline{S^n}$- $edg(X)$) is:

   $\underline{S^n}\text{-}\ edg(X) = X - \underline{S^n}(X).$

3. The supra external edge of X(denoted by$\overline{S^n} - edg(X)$) is:

   $\overline{S^n}\text{-}\ edg(X) = \overline{S^n}(X) - X.$

Example 2.2.
Let (U, R, S^n) be an supra approximation space S^nAS, where $U = \{a, b, c, d\}$, R be a binary relation on U with the neighborhoods defined as $N(a) = \{a, b\}$, $N(b) = \{b, c\}$,$N(c) = \{d\}$, and $N(d) = \{c\}$.
The supra topology$S^n(U)$ is given by:
$S^n(U) = \{\emptyset, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\} U\}.$
The complement of$S^n(U)$,denoted as $\left(S^n(U)\right)^c$, is:
$\left(S^n(U)\right)^c = \{\emptyset, \{a\}, \{c\}, \{d\}, \{a, b\} \{c, d\}, \{a, d\} \{a, b, d\}, \{a, b, c\}, U\}.$
Now, consider the sets $X = \{b, c\}$, $Y = \{a, b\}$, and $Z = \{b, c, d\}$.

1. For X

   The lower approximation $\underline{S^n}(X) = X.$
   The upper approximation $\overline{S^n}(X) = \{a, b, c\}.$
   The supra boundary $S^n\text{-}bnd(\mathrm{X}) = \{a\}.$
   The supra internal $\underline{S^n} - edg(X) = Y - \underline{S^n}(X) = \emptyset.$
   The supra external $\overline{S^n} - edg(X) = \overline{S^n}(X) - X = \{a\}.$

2. For Y

   The lower and upper approximation are equal $\underline{S^n}(Y) = \{a, b\} = Y = \overline{S^n}(Y).$
   Thus, supra boundary $S^n\text{-}bnd(\mathrm{Y}) = \emptyset$, and Y is S^n –definable.

3. For Z

   The lower approximation $\underline{S^n}(Z) = \{c, d\}.$
   The upper approximation $\overline{S^n}(Z) = U.$
   The supra boundary $S^n\text{-}bnd(\mathrm{Z}) = \{a, b\}.$
   The supra internal $\underline{S^n} - edg(Z) = Z - \underline{S^n}(Z) = \{a\}.$
   The supra external $\overline{S^n} - edg(Z) = \overline{S^n}(Z) - Z = \{b\}.$

Definition 2.7.
Let (U, R, S^n) be a supra approximation space S^nAS and $X \subseteq U$. Then, X is called:

1. S^n -internally definable if and only if $\underline{S^n}(X) = X$.

2. S^n -externally definable if and only if $\overline{S^n}(X) = X$.

From a topological perspective, $\underline{S^n}(X)$ and $\overline{S^n}(X)$, can be reinterpreted using topological concepts.

Definition 2.8. (El-Shafei et al., 2016; Mashhour, 1983; Talabeigi & Computing, 2022)
A subfamily μ of U is said to be a supra topological on U, if:

1. $\emptyset, U \in \mu$ and
2. If $B_i \in \mu$ for all $i \in j$, then $\cup B_i \in \mu$.

The pair(U, μ) is called a supra topological space.The elements of μ are called supra- open sets in (U, μ) and a subset of a supra topological space (U, μ) is called supra-closed set if its complement is a supra- open set.

Definition 2.9.
Let (U, R, S^n) be a supra approximation space S^nAS and a supra topological space.
Then, for every $X \subseteq U$ we have:

1. X is said to be S^n-internally (S^n-externally,S^n-totally) definable if and only if X is supra open (supra closed, supra clopen) set in a supra topological space,
2. X is said to be S^n –undefinable (rough) set if and only if X is neither supra open nor supra closed in the supra topological space. clarification.

Proposition 2.2.
Let (U, R, S^n) be a supra approximation space S^nAS, $X \subseteq U$,and R be an equivalence relation. Then:

1. $\underline{S^n}\left(\underline{S^n}(X)\right) = \overline{S^n}\left(\underline{S^n}(X)\right)$;
2. $\overline{S^n}\left(\overline{S^n}(X)\right) = \underline{S^n}\left(\overline{S^n}(X)\right)$.

3- Separation Axioms in Supra Approximation Spaces
The main purpose in this section, we define and study some new Separation axioms by define S^n-open set which play an important role in distinguishing between sets and points in a topological space, and we study some separation properties in a supra approximation space.

Definition 3.1.
A supra approximation space (U, R, S^n) is:

1. T_0 -space if and only if for every two distinct points$x$ and y on U, there exists an S^n-open set Gsuch that $x \in G$ and $y \notin G$,
2. T_1 -space if and only if for every two distinct points x and y on U, there exist two S^n-open sets G and H such that $x \in G$, $y \notin G$, and $y \in H$, $x \notin H$.

3. T_2 -space if and only if for every two distinct points x and y on U,there exist two disjoint S^n-open sets G and H such that $x \in G$ and $y \in H$.

Clearly $T_2 \Rightarrow T_1 \Rightarrow T_0$.

Proposition 3.1.
A supra approximation space$(U, R, S^n)$is T_0 if and only if
$\overline{S^n}(\{x\}) \neq \overline{S^n}(\{y\})$ for all $x \neq y$,where $x, y \in U$.

Proof:
Consider $x \neq y$. Then, there exists $G \in S^n(U)$ such that $x \in G$ and $y \notin G$.

1. There exists $U - G \in \left(S^n(U)\right)^c$, where $x \notin U - G$ and $y \in U - G$ if and only if $x \notin \cap \left\{U - G: U - G \in \left(S^n(U)\right)^c: \{y\} \subseteq G\right\}$ if and only if $x \notin \overline{S^n}(\{y\})$ but $x \in \overline{S^n}(\{x\})$.
   Thus, $\overline{S^n}(\{x\}) \neq \overline{S^n}(\{y\})$.
2. Consider $\overline{S^n}(\{x\}) \neq \overline{S^n}(\{y\})$, i.e., there exists $z \in U$ such that $z \in \overline{S^n}(\{x\})$ and $z \notin \overline{S^n}(\{y\})$.

   Suppose $x \in \overline{S^n}(\{y\})$. This implies $\overline{S^n}(\{x\}) \subseteq \overline{S^n}(\{y\})$, which further implies $z \in \overline{S^n}(\{y\})$.
   This leads to a contradiction. Therefore, $x \notin \overline{S^n}(\{y\})$.
   Hence, $U - \overline{S^n}(\{y\})$ is a supra set containing x but not y.
   Thus, S^n is a T_0 space.

Proposition 3.2.
A supra approximation space (U, R, S^n) is T_1-space if and only if $\{x\}$ is S^n-externaly definable, for all $x \in U$.

Proof:

1. Consider $x \in U$, S^nAS(U, R, S^n) be a T_1-space. To prove that $\{x\}$ is S^n-externally definable,
   we show that $\{x\}^c$ is S^n-internally definable (a supra set).
   Let $y \in \{x\}^c (i.e., y \neq x)$.Since S^nAS is a T_1-space, there exists $G, H \in S^n(U)$ such that $y \in G, x \notin G$ and $x \in H, y \notin H$.
   Thus, for all $y \in \{x\}^c$ there exists $G \in S^n(U): y \in G \subseteq \{x\}$.
   Since y is arbitrary, $\{x\}^c = \cup \{G \in S^n(U), G \subseteq \{x\}^c\}$.
   Therefore, $\{x\}^c$ is S^n-open, and $\{x\}$ is S^n- externally definable.
2. For the inversions consequence, assume $\{x\}$ be S^n-externally definable i.e., $\{x\}^c$ be a S^n-open set.

Consider $x, y \in U$ such that $x \neq y$. This means:
1. $x \in \{y\}^c, y \notin \{y\}^c$;
2. $y \in \{x\}^c, x \notin \{x\}^c$;
since $\{x\}^c$ and $\{y\}^c$ are S^n-open sets, a S^nAS (U, R, S^n) is a T_1- Space.

Proposition 3.3.
Consider (U, R, S^n) is S^nAS where R is preordering relation, if (U, R, S^n) is a T_1-Space then X is S^n- definable for all $X \subseteq U$.

Proof:
Consider $X \subseteq U$ to prove X is S^n-definable, we show that X is both S^n-internally and S^n-externally definable.

1. Since S^nAS is aT_1- Space,$\{x\}$ is S^n- externally definable for all $X \subseteq U$ (from Proposition3.2).
2. Both $X$and $U - X$ can be written as:

$$X = \cup_{x \in X} \{x\},$$
$$U - X = \cup_{x \in U-X} \{x\}.$$

3. from Proposition 3.7, X and $U - X$ are S^n-externally definable sets.
4. Since $U - X$ are S^n-externally definable, X is S^n-internally definable.
5. Thus,X is both S^n-internally and S^n-externally definable, making it S^n- definable.

Proposition 3.4.
A S^nAS that is a T_1- Space with preordering relation is a discrete approximation space.

Proof:
This follows directly from part 2 of Definition 3.1.

Definition 3.2.
Let (U, R, S^n)be S^nAS. The relation T_2 on U is defined by:
xT_2y if and only if there exists $G, H \in S^n(U)$ such that $x \in G, y \in H$ and $G \cap H = \emptyset$.
This relation is renamed (a separating relation).

Definition 3.3.
Let (U, R, S^n)be an S^nAS, and let T_2 be a separating relation on U. Then
$X^{T_2} = \{ y \in U: yT_2x, \forall\, x \in X\}$ is called the separating set of X.

Definition 3.4.
Consider (U, R, S^n)be an S^nAS and let $X, Y \subseteq U$. Then,
XT_2Y if xT_2y for all $x \in X, y \in Y$.

Remark 3.2.
We observe that the separating relationT_2 is has the following properties:

1. Irreflexive relation: xT_2y if and only if $x \neq y$.

2. Symmetric relation: xT_2y if and only if yT_2x.

For the $\Psi \subseteq S^n(U)$, where Ψ be collection of all neighborhoods, the relation T_2 can be redefined as follow:
xT_2y if and only if there exists $N_1, N_2 \in \Psi,$ $\mathrm{x} \in N_1, y \in N_2, N_1 \cap N_2 = \emptyset$.

Proposition 3.5.
Let (U, R, S^n)be an S^nAS, and let $X \subseteq U$.Then:

1. $X^{T_2} \subseteq X^C$.
2. $X^{T_2} \cap X = \emptyset$

Proof:

1. $y \in X^{T_2}$ if and only if yT_2x for all $x \in X$, which implies $y \neq x$ for all $x \in X$. if and only if $y \in X^C$.

2.Assume $X^{T_2} \cap X \neq \emptyset$ if and only if there exist $y \in U$ such that $y \in X^{T_2}$ and $y \in X$ implies $y \in X^C$ if and only if $X^c \cap X \neq \emptyset$ (contradiction).Then $X^{T_2} \cap X = \emptyset$.

Proposition 3.6.
Consider (U, R, S^n)be an S^nAS and $X \subseteq U$.Then:

1. $X^{cT_2} \subseteq X$.
2. $X \subseteq X^{T_2c}$.

Proof:
This follows directly from Proposition3.5.

Proposition 3.7.(Lin, 1988, 1989; Lin & computing, 1997)
Consider (U, R, S^n)be an S^nAS and $X \subseteq U$.Then, X^{T_2} is a supra set.

Proof:
To prove that X^{T_2} is a supra set, it is suffices to show that X^{T_2} can be written as a union of supra set.

1. Let $y_1 \in X^{T_2}$. By Definition 3.3, this means $y_1T_2x,$ for all $x \in X$.
2. From Definition 3.2, there exists$G_1, H_1 \in S^n(U)$ such that $y_1 \in G_1, x \in H_1$and $G_1 \cap H_1 = \emptyset$.
3. Since $G_1 \cap H_1 = \emptyset,$ for all $x \in X$ (where $x \in H_1),$ it follows that $G_1 \subseteq X^{T_2}$.
4. Thus $y_1 \in X^{T_2}$ implies there exists$G_1 \in S^n(U)$ such that $y_1 \in G_1 \subseteq X^{T_2}$.

similarity, for any $y_2 \in X^{T_2}$, there exists $G_2 \in S^n(U)$ such that $y_2 \in G_2 \subseteq X^{T_2}$.
Finally, $y_i \in X^{T_2}$ implies $\exists G_i \in S^n(U)$ such that $y_i \in G_i \subseteq X^{T_2}$. Hence

$\cup_{i=1}^{n}\{y_i\} \subseteq \cup_{i=1}^{n} G_i \subseteq X^{T_2} = \cup_{i=1}^{n}\{y_i\}$ i.e., $X^{T_2} = \cup_{i=1}^{n} G_i$.Then X^{T_2} is a supra set.

Example 3.1.
Let (U, R, S^n) be an supra approximation space S^nAS, where $U = \{a, b, c, d\}$ and the neighborhoods are defined as:

$N(a) = \{a, b\}$,
$N(b) = \{b\}$,
$N(c) = \{d\}$,
$N(d) = \{a, c\}$.

The supra set and its complement are:

$$S^n(U) = \{\emptyset, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{a, b, c\}, \{a, c, d\}, U\},$$
$$\left(S^n(U)\right)^C = \{\emptyset, \{b\}, \{d\}, \{c\}, \{a, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, U\}.$$

Table (1). Table of operators.

X	X^{T_2}	$(X^{T_2})^c$	X^{cT_2}	$\underline{S^n}(X)$	$\overline{S^n}(X)$
$\emptyset$	U	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{a\}$	$\{b, d\}$	$\{a, c\}$	$\emptyset$	$\emptyset$	$\{a, c\}$
$\{b\}$	$\{a, c, d\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$
$\{c\}$	$\{b, d\}$	$\{a, c\}$	$\emptyset$	$\emptyset$	$\{c\}$
$\{d\}$	$\{a, b, c\}$	$\{d\}$	$\{d\}$	$\{d\}$	$\{d\}$
$\{a, b\}$	$\{d\}$	$\{a, b, c\}$	$\{b\}$	$\{a, b\}$	$\{a, b, c\}$
$\{a, c\}$	$\{b, d\}$	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$
$\{a, d\}$	$\{b\}$	$\{a, c, d\}$	$\{d\}$	$\{d\}$	$\{a, c, d\}$
$\{b, c\}$	$\{d\}$	$\{a, b, c\}$	$\{b\}$	$\{b\}$	$\{a, b, c\}$
$\{b, d\}$	$\{a, c\}$	$\{b, d\}$	$\{b, d\}$	$\{b, d\}$	$\{b, d\}$
$\{c, d\}$	$\{b\}$	$\{a, c, d\}$	$\{d\}$	$\{d\}$	$\{c, d\}$
$\{a, b, c\}$	$\{d\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$
$\{a, b, d\}$	$\emptyset$	U	$\{b, d\}$	$\{a, b, d\}$	U
$\{a, c, d\}$	$\{b\}$	$\{a, c, d\}$	$\{a, c, d\}$	$\{a, c, d\}$	$\{a, c, d\}$
$\{b, c, d\}$	$\emptyset$	U	$\{b, d\}$	$\{b, d\}$	U
U	$\emptyset$	U	U	U	U

Remark 3.3.
By comparing the lower approximation operation $\underline{S^n}(X)$ and X^{cT_2}, it follows that $\underline{S^n}(X)$ provides the best lower approximation. This can be observed in the sets$\{a, b\}$, , $\{a, b, d\}$ from the previous example. Similarly, by comparing the upper approximation $\overline{S^n}(X)$ and $(X^{T_2})^c$,it follows that $\overline{S^n}(X)$ provides the best upper approximation, as seen in the sets $\{c\}$ and $\{c, d\}$.

Proposition 3.8.
A (U, R, S^n) be a supra approximation space (S^nAS). Then, (U, R, S^n) is a T_2-Space if and only if $X^{T_2} = X^c$, for all $X \subseteq U$.

Proof:

From Proposition 3.5 we have $X^{T_2} \subseteq X^c$ for all $X \subseteq U$. Now, let $y \in X^c$. Then, $y \neq x$, for all $x \in X$,which implies there existence of $G, H \in S^n(U)$ such that $x \in G, y \in H$, and$G \cap H = \emptyset$ This means yT_2x for all $x \in X$, which is equivalent to $y \in X^{T_2}$. Therefore, ,$X^c \subseteq X^{T_2}$, and we conclude that $X^{T_2} = X^c$.

Proposition 3.9.(Al-Shami, 2016, 2017)
Let (U, R, S^n) be a supra approximation space (S^nAS). and $X \subseteq U$.Then, X is S^n-definible if and only if $(X^{T_2})^c = (X^c)^{T_2}$.

Proof:

1. **First part:** Assume $X \subseteq U$ is S^n-definible. Let $y \neq x$ where $x \in X$ and$y \in X^c$. Since X and X^c are supra sets, xT_2y for all $x \in X, y \in X^c$. By Definition 4.3, we have $y \in X^{T_2}$and $x \in (X^c)^{T_2}$ which implies $X^{T_2} = X^c$. Therefore $(X^{T_2})^c = X = (X^c)^{T_2}$.
2. **Second part:** Assume $(X^{T_2})^c = X = (X^c)^{T_2}$. By proposition3.7, if $(X^c)^{T_2} = X$. Then X is a supra set .Additionally, if $(X^{T_2})^c = X$ and X^c be a supra set, then X be S^n-definable set.

Proposition 3.10.
Let (U, R, S^n) be a S^nAS and $X \subseteq U$. The following statements are equivalent:

1. S^nAS is T_2-Space;
2. X is S^n-definable for all $X \subseteq U$;
3. S^nAS is discrete approximation space.

Proof:

1. $1 \Longrightarrow 2$: Since (U, R, S^n) is a T_2-Space, from Proposition 3.8, we have $(X^c)^{T_2} = X$ and $X^{T_2} = X^c$, i.e., $X^{T_2 c} = X$ and $X^{c T_2} = X$. Then, X is an S^n-definible set from Proposition 3.9.
2. $2 \Longrightarrow 3$: This is Obvious.
3. $3 \Longrightarrow 1$: Let $\{x\}$ and $\{y\}$ are disjoint S^n-definable sets such that $\{x\} \cap \{y\} = \emptyset$, with $x \in \{x\}$ and $y \in \{y\}$ for all $x, y \in U$ Then S^nAS is T_2-Space.

Proposition 3.11.(Yao, 1999)
Let (U, R, S^n) be a S^nAS and $X \subseteq U$. If R is an equivalence relation, then $X^{T_2} = \left(\overline{S^n}(X)\right)^c$.

Proof:
Ago R is an equivalence relation, xT_2y if and only if $N_x \cap N_y = \emptyset$ (from Proposition 3.9).

1. $y \in X^{T_2}$ if and only if yT_2x, for all $x \in X$,

2. if and only if $N_x \cap N_y = \emptyset$,
3. if and only if $x \notin N_y$ for $x \in X$,
4. if and only if $X \cap N_y = \emptyset$,
5. if and only if $y \in \left(\overline{S^n}(X)\right)^c$.

Thus, $X^{T_2} = \left(\overline{S^n}(X)\right)^c$.

CONCLUSION

In this work, we introduced and studied a new space based on a generalized neighbourhood system by using the concept of supra topology, called a supra approximation space (briefly S^nAS). We investigated key properties of S^nAS and compared its advantages with those of the neighbourhood approximation space.

Additionally, we defined and analyzed new Separation axioms using S^n-open sets, which play a crucial role in distinguishing between sets and points in a topological space. We also explored separation properties within the supra approximation space. In future work, we aim to extend the application of S^nAS to near- open sets and near- closed sets, further expanding its theoretical and practical implications.

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